Nominalism and Constructivism in Seventeenth-Century Mathematical Philosophy

David Sepkoski

Routledge Studies in Seventeenth-Century Philosophy

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What was the basis for the adoption of mathematics as the primary mode of discourse for describing natural events by a large segment of the philosophical community in the seventeenth century?

In answering this question, this book demonstrates that a significant group of philosophers shared the belief that there is no necessary correspondence between external reality and objects of human understanding, which they held to include the objects of mathematical and linguistic discourse. The result is a scholarly reliable, but accessible, account of the role of mathematics in the works of (amongst others) Descartes, Gassendi, Hobbes, Newton, and Berkeley.

This impressive volume will benefit scholars interested in the history of philosophy, mathematical philosophy and the history of mathematics.

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11 **Nominalism and Constructivism in Seventeenth-Century Mathematical Philosophy**
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Contents

Acknowledgements viii

Introduction: Mathematization and the 'Language of Nature' 1

1 Realists and Nominalists: Language and Mathematics before the Scientific Revolution 11

2 Ontology Recapitulates Epistemology: Gassendi, Epicurean Atomism, and Nominalism 33

3 British Empiricism, Nominalism, and Constructivism 54

4 Three Mathematicians: Constructivist Epistemology and the New Mathematical Methods 83

Conclusion: Mathematization and the Nature of Language 124

Notes 128
Bibliography 154
Index 165

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Introduction: Mathematization and the ‘Language of Nature’

The topic of the ‘mathematization of nature’ the process by which natural phenomena came to be understood more and more as representable by universal, quantifiable mathematical laws is of paramount importance in understanding the historical development of modern Western science. With roots in the mathematical astronomy and physics of the ancient Mediterranean, mathematization is generally understood to have played out over centuries, from the discovery of laws of hydrostatics and the articulation of a theory of conic sections in Hellenistic Greece, through investigations into the properties of motion in medieval Europe, to the Renaissance of mathematical astronomy of Copernicus and Kepler, finally concluding in the unified principles of mathematical physics of René Descartes, Isaac Newton, and G. W. Leibniz. This pattern of development has provided, in some interpretations, evidence for the positive accumulation of scientific knowledge over time.[1] Others have characterized the history of mathematization as a series of fits and starts, with advancement spurred by the ‘re discovery’ of ancient texts and cosmologies.[2] Most interpretations, however, have agreed on an essential feature of the trend towards mathematization: as natural philosophers particularly in the sixteenth and seventeenth centuries gained more confidence in the predictive accuracy and explanatory power of their mathematics, so too did their certainty grow that mathematics expressed an underlying truth about the natural world that the universe *is*, in some real sense, mathematical. Or, in the famous words of the arch mathematizer, Galileo:

> Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures, without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth.[3]

This statement is normally interpreted to exemplify the early modern trope of mathematics as a ‘language of nature’; Galileo’s conception has in the past been cited as the metaphysical basis for the development of mathematized physics in the Scientific Revolution.[4] A question that has seldom been asked, however, inquires into the human faculty required for such a task: if indeed mathematics is the ‘language’ of nature, what does Galileo think is involved in the act of ‘reading’ its characters? We should not assume that such an apparently basic activity as reading is conceptually unproblematic from a historical standpoint, since it calls into question conventions in grammar, logic, and epistemology that are certainly contingent. Although striking, Galileo’s claim has been echoed throughout the history of philosophy; mathematics was, and indeed, is, quite often referred to as a ‘language,’ and many philosophers have implied that it is the language best suited for expressing a ‘natural order.’ A deeper question, however, asks what was the philosophical basis for language, and how, at different times and in different philosophical traditions, was the idea of ‘reading’ (and writing) the mathematical language of nature actually construed? The epistemology of mathematization is fundamentally linked to the epistemology of language.

In fact, mathematization was part of a larger discussion about the nature of linguistic and mental representation that is as old as Western mathematics. This debate asks a number of deeply important and associated epistemological questions: How reliable are our ideas or representations of natural objects? Are ideas objects themselves, or merely figments or phantasms? What is the referential content of words in spoken or written language? Are words conventional definitions or ‘real’ things? Can universal or essential attributes of objects be perceived, or only individual qualities? Are the objects of mathematics part of nature, or of the mind? Do mathematical representations reflect some necessary order in the universe, and are mathematical objects ‘real’? The terms under which these questions were considered varied greatly according to the broader theological and philosophical issues and conventions of the particular historical periods in which they were addressed. Thus, for example, medieval arguments over the existence of real universals were oriented toward important theological debates about the nature of the sacrament, while seventeenth century arguments in a similar vein were more concerned with establishing a reliable basis for a mathematical and experimental natural philosophy. But in all instances, an important lesson is clear: natural philosophers in pre modern Europe did not consider questions about the nature of mathematics to be separate from concerns about the basis for language (and logic). Up to and through the time of Newton, mathematics *was* philosophy.

The conflict between two great traditions in the history of philosophy from antiquity through the Scientific Revolution has been characterized by one scholar as “the battle of the Gods and Giants.” According to Thomas Lennon, it consists of

> realism on universals (versus nominalism), a tendency at least towards idealism (versus relativism), a similar tendency towards monism (versus

> pluralism), necessitarianism (versus tychism), plenism (versus atoms and the void), the rationalist's emphasis on reason and the *a priori* (versus the empiricist's emphasis on experience and the *a posteriori*).[5]

The valorization of mathematical physics in the late seventeenth century has been read as the triumph of the Gods—the implicit or explicit vindication of the belief that mathematics and its objects exist in (or have reference to) external reality. This book, however, will challenge that received view. I will argue that the tradition of nominalism—which argued against the existence of perceivable real universals—struck an important counterpoint to realist/essentialist philosophy of mind and mathematics which, beginning with the great nominalist theologians of the late Middle Ages, constituted an active resistance to the universalizing tendencies of contemporary epistemologies. Nominalism has been connected by scholars to important philosophical and scientific developments in the seventeenth and eighteenth centuries, particularly in relationship to theological and biological questions.[6] It has not, however, been widely recognized as a factor in early modern mathematical practice and philosophy.[7] This book will argue, nonetheless, that nominalistic strategies provided early modern philosophers, including Pierre Gassendi, Thomas Hobbes, and George Berkeley, with an important set of arguments for describing the ontological content of mathematical entities and magnitudes in a way that departed from Platonic realist accounts without sacrificing reference to empirical experience. Furthermore, I will argue that this nominalistic approach was closely linked to the development of a 'constructivist' philosophy of mathematics, which has been connected by a number of scholars to the geometric and analytic practices of mathematicians such as Isaac Barrow and Isaac Newton. More generally, I hope to demonstrate the close relationship between mathematical philosophy and wider philosophical and theological beliefs during the Scientific Revolution, and to reinforce the historiographical point that, at least during the early modern period, the history of mathematics was inextricably linked with contemporary currents in broader intellectual culture.

* * *

In the middle decades of the twentieth century, historians of sixteenth- and seventeenth-century mathematical sciences tended to focus on the conceptual or metaphysical foundations of mathematical natural philosophy. This approach, exemplified in the work of scholars such as E. A. Burtt, Alexandre Koyré, and E. J. Dijksterhuis, had a major role in defining the period of time now traditionally known as the 'Scientific Revolution.' While this conceptualist program has been eclipsed in more recent decades by studies of the social, institutional, and technical context of early modern mathematical practice, there remains a strong tradition in intellectual history of mathematics. Recent scholarship by Paolo Mancosu, Douglas Jesseph, Niccolo Guicciardini, Michael Mahoney, and Peter Dear has analyzed intellectual traditions in mathematical practice, and shown that seventeenth century mathematical philosophy developed out of a more complex and heterogeneous web of influences than was envisioned by those earlier scholars.[8]

However, one liability in the greater degree of specialization with which historians have approached the history of early science has been a fragmentation of the older, more unified narratives. This is particularly the case in the history of mathematics. While we now have a better understanding of the diversity of mathematical practices, methods, and traditions in the sixteenth through eighteenth centuries, there is a need to connect the history of mathematics with a broader view and with attention to its general philosophical context. This need is more than simply historiographical: early modern natural philosophers themselves did not separate mathematical and scientific pursuits from more general questions in philosophy, so understanding the philosophical basis of their beliefs gives important insight into the development of contemporary mathematical natural philosophy.

Despite changes in historiography over the past seventy five years, historians still universally acknowledge that a change of vital importance occurred in the mathematical sciences during the period that has traditionally been called the 'Scientific Revolution.' Beginning roughly with the publication of Copernicus' *De revolutionibus* in 1543 and concluding in the early eighteenth century with the general acceptance of Newton's new mechanics in the *Principia*, this period saw an explosion of innovations both in physics and 'pure' mathematics, including the development of techniques in analytic geometry that led to the discovery of the calculus.[9] A central problem for historians of early modern science, then, has been to account for the transformation of mathematics and mathematical science during this period, and to situate the developments in mathematics within a contextual understanding of contemporary scientific and philosophical issues.

This, generally speaking, is the concern of the present study: to give an account of philosophical debates that bore directly on the construction of beliefs about the reality, usefulness, and certainty of mathematical demonstrations and objects in seventeenth century natural philosophy, and to shed light on the complex web of factors that contributed to the formulation of 'modern' mathematical science. Where many other studies have focused primarily on the development of technical and practical issues in the history of mathematics, I will concentrate rather on the philosophical basis for mathematical practice, which was clearly elaborated in a number of epistemological statements in the works of seventeenth century natural philosophers. I will argue that a crucial concept in the philosophy of early modern mathematics was the convergence of linguistic and mathematical theories of representation that led some contemporary philosophers to regard mathematics as a language for expressing regularities observed in nature, subject to the conventions of natural and artificial languages. The seventeenth century saw a wide range of beliefs concerning the epistemological status of language, and I will focus mainly on one particular theme: the nominalist tradition, which included the work of Pierre Gassendi, Thomas Hobbes, and George Berkeley, the latter of whose ‘constructivist’ approach to language and mathematics influenced the development of a formalist approach to mathematics and mathematical philosophy.[10] The unifying factor among these philosophers was a concern not simply with the rules of mathematical practice, but with a general epistemological account of representation that viewed objects of human knowledge both in mathematics and language as ‘constructed’ out of distinct experiences, rather than as innately perceived archetypes in a divine order.

* * *

At the heart of this problem is the question of how natural philosophers justified claims about knowledge of the physical world. It is commonly accepted by historians that one of the fundamental aspects of the Scientific Revolution and the development of ‘modern’ science was the increasing acceptance of mathematics as the ‘proper’ language for expressing natural laws and regularities. Mathematization arose as a central concept in historical analysis in the earliest years of the history of science as a formal discipline, and although it has undergone many revisions, it still retains currency today. The first important analyses of the metaphysics of mathematization are the works of Burtt, Koyré, and Dijksterhuis, who collectively helped found the ‘great tradition’ in pre World War II historiography of the Scientific Revolution.[11] Burtt summed up the role of mathematization as the establishment of “a new metaphysic a mathematical interpretation of the universe to furnish the final justification for the onward march of mechanical knowledge,” which he maintained characterized the philosophical outlook of scientists from Galileo through Newton.[12] He alleged that the transformative aspect of the Scientific Revolution in the physical sciences lay in the metaphysical, rather than the purely practical or technical, basis for describing nature in mathematical terms. According to Burtt, medieval cosmology was based on an ontological assumption that the structure of the universe reflected immanent properties of God; the ‘new’ science, on the other hand, substituted the rules of Euclidean and Archimedean geometry for the qualitative physics of Aristotle, and reduced the arbitrary structure of the medieval cosmos to a system which operated according to fixed laws and the rational rules of mathematics. Mathematics, in other words, became the ‘ethos’ of the Scientific Revolution, and practitioners such as Copernicus, Kepler, and Galileo sought mathematical descriptions because, according to their metaphysics, the universe must be organized according to a mathematical harmony.[13] This historical interpretation necessarily privileges the *a priori* aspect of mathematical science above the empirical, and Burtt maintains that Galileo and Descartes pursued mathematical descriptions as *a priori* insights into the rational order of creation.

In more recent years, historians have raised a number of objections to Burtt's analysis of the metaphysical basis for mathematics, and have challenged the notion that a single, paradigmatic 'metaphysic' of Platonism/Pythagoreanism underwrote the ontologies of most sixteenth- and seventeenth-century mathematical authors. Gary Hatfield, for example, takes exception with Burtt's definition of metaphysics as 'unconscious' first principles, noting that most major natural philosophers of that time quite consciously pursued metaphysical questions in their works. Among these figures, Hatfield concludes, only Kepler truly adopted Platonic and Pythagorean metaphysics, and Hatfield denies that Galileo even developed what can properly be called metaphysical themes.[14] This relates to an earlier criticism made by Edward W. Strong (one of Burtt's first critics). Strong claimed that sixteenth- and seventeenth-century mathematics does not show the influence of previous metaphysics, and he characterized 'metaphysical mathematics' as "the archaic and scientifically divorced tradition" revived by Zamberti, Domenico, Pico, Dee, and others in the sixteenth century, which he concludes did not figure in the development of 'modern' mathematical physics.[15] A common theme among Burtt's critics is a disdain for his understanding of metaphysics as 'unconscious presuppositions,' which implies that metaphysics is somehow outside of or beneath 'scientific' philosophy. Lorraine Daston explains that Burtt's arguments were shaped by his anti-positivism, and that his intent may have been to comment on his contemporaries—such as the emerging logical positivists—whom Burtt believed put too much faith in the certainty of mathematical logic. By exposing its historical development, Daston argues, Burtt hoped to expose the 'metaphysical core' of mathematical science, and thus to de-stabilize it, but this produced, in Daston's words, a "niggardly view of metaphysics . . . [that] led him into anachronism."[16]

Ultimately, however, it is Burtt's formulation of the role of 'unconscious' metaphysical beliefs in the history of the mathematical sciences that scholars have attacked, and not his suggestion that metaphysical (or ontological) principles figured in the development of early modern mathematical natural philosophy. This suggests that a conceptual or intellectual approach to the history of early modern mathematics is certainly viable, although some of the broad assumptions of Burtt, Koyré, Dijksterhuis, and others should be avoided. Rather than painting the history of the conceptual development of mathematics during the Scientific Revolution with broad strokes, close attention must be paid to the great variety of epistemological and ontological positions that featured in mathematico-philosophical beliefs of that period. Mathematization, as a central epistemological theme in the seventeenth century, continues to have currency for historians, but more recently scholars have highlighted the range of philosophical issues it raised for its practitioners.

Michael S. Mahoney has noted the "paradoxical" fact that just as more powerful mathematizing tools were being developed in the mid-seventeenth century, new questions were also being raised about the ontological basis for mathematical demonstrations. Mahoney comments that "the new analysis pointed mathematics away from physical ontology . . . enhanc[ing] its explanatory power over nature by moving conceptually beyond the intuitive limits of the physical world."[17] Michel Blay has similarly characterized a shift in mid seventeenth century mathematical natural philosophy away from "ontologico geometrical" realism towards a more analytical "mathematization" that "put aside any meaningful ontological claims it might have had" earlier.[18] This interpretation echoes Jacob Klein's conclusion in his classic study *Greek Mathematical Thought and the Origin of Algebra* that

> the manner in which the founders of modern science set about attaining a mathematical comprehension of the world's structure betrays, from the outset, a different conception of the world, a different understanding of the world's being . . . which rests upon a new ontological understanding.[19]

And Paolo Mancosu has shown that questions about the representative value of mathematics extended back into the sixteenth century, to a debate over the *Quaestro de certitudine mathematicarum* in which a number of Italian mathematicians (Piccolomini, Bianciani, etc.) questioned whether mathematics fit the Aristotelian model of demonstrative science. Mancosu argues that these debates influenced the views of seventeenth century natural philosophers, including Hobbes, Descartes, and Barrow, and that they centered around the status of different kinds of mathematical objects (i.e. geometrical figures versus algebraic symbols). The earlier tradition of the *Quaestro* was important in defining attitudes towards the mathematization of natural phenomena in the seventeenth century, and Mancosu emphasizes the degree to which many of the "central mathematical concepts" were not well defined in the seventeenth century.[20]

The philosophical problems associated with the rising prominence of analytic methods in seventeenth century British mathematics are a central focus of Helena M. Pycior's work, and her analysis has particularly influenced the present study.[21] In surveying arguments for and against analysis and traditional geometry in the works of British authors from William Oughtred and Thomas Harriot through Berkeley and the Scottish mathematical school of the mid eighteenth century, Pycior has emphasized that general philosophical questions concerning the nature of representation played a major role in shaping mathematical philosophies. She particularly draws attention to the degree to which the debate between proponents of synthetic and analytic methods "forced British thinkers to reflect long and hard on symbolical reasoning," and she characterizes British philosophers of this period including Hobbes, Wallis, Barrow, Newton, and Berkeley as "a 'test group' for the symbolical reasoning and later semiotics that would in the nineteenth and twentieth centuries so change not only mathematics but logic, philosophy, and language studies as well."[22] This last statement aptly summarizes many of the central concerns of the present book: I will investigate whether questions about mathematics and mathematization during the Scientific Revolution also engaged problems in the philosophy of logic and language, and will construct a picture of mathematical natural philosophy during this period that takes these factors into full account.

Although I will focus in large part on many members of the central group of British mathematical thinkers that Pycior highlights, I will turn also to continental influences, and particularly to Gassendi, whose philosophy, I argue, was a locus for questions about the relationship of mathematics to issues in linguistic and logical representation. My choice of Gassendi who does not figure prominently in any of the studies I have mentioned also partly determines my emphasis on nominalism as a tradition in epistemology, and this book is the first, to my knowledge, to seriously consider the influence of the nominalist tradition on seventeenth century mathematical thought. It should be emphasized, however, that I argue that nominalism (and the epistemological constructivism which I will argue accompanies it) was only one of a number of important conceptual approaches to mathematics in the seventeenth century. A lesson I draw from the reaction to Burtt and Koyré is the danger in characterizing conceptual developments in mathematics (or any science) using a single philosophical or metaphysical label, and while I hope to draw attention to the importance of nominalism in relation to mathematics, I will not attempt to argue that nominalism was the pre eminent influence, or to minimize the importance of other philosophical approaches to mathematics that were prominent during this period.

* * *

One of the most important revisions to recent historiography of science has been the recognition of an early modern tradition in which scientific knowledge was understood not in an idealistic Platonic sense to be reflection of eternal truths, but rather as an 'artificial' production of human ingenuity. In sociology and the social history of science, this has been termed 'maker's knowledge,' and attention has been paid particularly to those practitioners Francis Bacon and Robert Boyle, most prominently who presented scientific findings as deliberate 'artifice.'[23] In the history of philosophy and of mathematics, this tradition has a parallel in what Dear and others have described as the constructivist approach to mathematics: the development of the notion that mathematical demonstrations (especially in geometry) "were things *to be drawn* rather than pre existing in a Platonic realm."[24] Dear has focused primarily on the articulation of beliefs about the legitimacy of mathematics in the late Aristotelian hierarchy of the sciences and in his account mathematics gained stature, beginning in the late sixteenth century, because of arguments that promoted its properly 'demonstrative' character. Others have made even broader claims for this kind of 'epistemological constructivism': Amos Funkenstein has argued that in the early modern period "a new ideal of knowledge was born the idea of knowledge by doing, or knowledge by construction."[25] According to Funkenstein's analysis, this constructivist ideal was born in the medieval nominalist philosophy of William of Ockham, Duns Scotus, Jean Buridan, Walter Burley, and others. It was transferred to the modern period primarily through the vehicle of theology: the concomitant of nominalist ontology was voluntarist theology, which emphasized the independence of God's ordained and absolute powers. In other words, voluntarism is a theology that respects God's absolute omnipotence, including His ability to intervene in and alter the established course of nature. According to Funkenstein's provocative (and occasionally overly ambitious) argument, many key seventeenth century natural philosophers were voluntarists, and therefore *must have shared* an allegiance to nominalist metaphysics with their medieval forbears.

Funkenstein was not a historian of mathematics, so unfortunately his claims about the significance of the nominalist/voluntarist tradition for the history of mathematization are underdeveloped. However, developing these claims is a major subtext of this book. Funkenstein drew attention to the significance of the medieval interest in constructing 'counterfactual' or hypothetical worlds in order to test the limits of God's absolute powers. This practice, which attracted Ockham, Nicole Oresme, and other fourteenth century nominalists, often involved complex logical arguments about the physical properties of space and time, and as such have a fairly clear scientific resonance. What Funkenstein does not discuss, however, is the even more striking parallel in the practice of seventeenth century practitioners of mathematics such as Newton's colleague and mentor, Isaac Barrow of discussing counterfactual *mathematical* universes in which God's absolute power might invalidate the established principles of geometry and arithmetic. I will argue that just as these counterfactual realities were no idle game for fourteenth century nominalists (they, in fact, related to doctrinal debates concerning God's corporeality that were anything but trivial) mathematical counterfactuals in the seventeenth century allowed certain mathematicians to explore the very legitimate (in their minds) possibility that mathematics was not an indelible language of nature. In particular, this later mathematical context addressed the theological question of whether God was bound by mathematical logic as many philosophers believed he was bound by logical principles such as identity and non contradiction.

This question also relates to the epistemology of nominalism, which in the seventeenth century was expressed most forcefully by the French natural philosopher Pierre Gassendi and the exiled Englishman Thomas Hobbes. Both were drawn to explore ontological questions about language and mathematics, and their nominalism influenced discussions of both categories of representation. Hobbes was the more active mathematical practitioner, but Gassendi developed a more thoroughgoing philosophical system of nominalism and indeed articulated a coherent nominalist account of mathematics. Though Barrow and his friend Newton were not the avowed nominalists Hobbes and Gassendi were, they read and appreciated Gassendi, and shared his voluntarist theology. Where the mathematical practice of Newton and Barrow meets the nominalist philosophy of Gassendi and Hobbes is in epistemological constructivism: Gassendi's rigorous program of through and through epistemic constructivism is reflected in Barrow's (and, to a lesser extent, Newton's) more moderate mathematical constructivism. This connection will be the subject of the concluding chapter of the book.

Of course, this trajectory is only significant if it leads towards some clarification of the emergence of modern mathematical philosophy. The final (and most ambitious) claim if this book is that nominalist philosophy of representation informs the rise of an instrumentalist and even formalist approach to mathematics. Indeed, nominalism and constructivism raised questions about the ontological reality of mathematical objects even as it contributed to the pre eminence of mathematical physics by the close of the seventeenth century. The results can be seen in the disputes during the early eighteenth century over the foundations of the calculus which brought English and French 'Newtonians' into conflict with, on the one hand, the followers of Leibniz, and on the other, with the Irish bishop and natural philosopher George Berkeley. The resolution of this conflict (which this book will invoke but not examine) saw the development of a formal approach to mathematics which sidestepped the theological and metaphysical issues central to the nominalist critique.

1 Realists and Nominalists: Language and Mathematics before the Scientific Revolution

While much of medieval and Renaissance grammar, logic, and metaphysics was based on interpretations of classical texts, it would be a mistake to assume that the texts of Plato and Aristotle that were read in the thirteenth, fourteenth and fifteenth centuries were equivalent to those we recognize as genuine today.[1] A number of spurious texts were included among the canonical works of Aristotle, and discrepancies in translations and manuscript editions of texts often resulted in variations among editions of the same work. Furthermore, as I will discuss in the next section, the ‘Aristotelian’ philosophy of scholastic authors such as Thomas Aquinas should not be equated with the philosophy of Aristotle himself, since medieval authors’ interpretations drew on a wide variety of sources, and in many cases went to great lengths to reconcile Greek philosophy with Christian doctrine, often at the expense of fidelity to the original texts. This is particularly an issue when confronting sixteenth and seventeenth century critiques of ‘Aristotelian’ philosophy, which are often directed more at scholastic interpretations than at the actual teachings of Aristotle.

It is still true, however, that most of the central issues in epistemology over the past two millennia received their first systematic treatment by either Plato or Aristotle, and that later authors both realist and nominalist consciously built on these foundations. The positions produced in classical antiquity that we are concerned with can be grouped roughly as follows: (1) the actual teachings of Plato and Aristotle; (2) independent mathematical works by technical authors such as Apollonius, Euclid, Archimedes, and others; (3) commentaries on either Plato or Aristotle, such as Proclus’ highly influential Platonic reading of Euclid; and (4) reactions against Plato and/or Aristotle, including those of the Epicureans, Stoics, and the Academic and Pyrrhonian skeptical schools. This grouping reflects those works or philosophical movements that were most directly influential during the Renaissance and Scientific Revolution, and my sketch here is mostly for the purpose of contextualizing the later portions of this study. It is not intended as a comprehensive survey of classical epistemology.[2]

The Philosophy of Language and Mathematics in Antiquity

Greek authors' interest in epistemological questions about language originated at roughly the same time as early speculations about cosmology and mathematics, most notably in the writings of the pre Socratic authors Parmenides, Protagoras, and Prodicus. This early study was mostly concerned with the origin and etymology of language, but certain authors, such as Parmenides, actually developed a more comprehensive theory of signification, centered on the proper application and interpretation of 'names.'[3] This theory, which was later modified by Protagoras and Prodicus, assumed a correspondence between names and objective reality, and thus may be seen as a forerunner to Plato's idealistic treatment of language. Plato takes up this issue in the dialogue *Cratylus*, where he contrasts the notion that language has arbitrary conventional signification with the belief that words themselves somehow embody the essence of their referents through similitude or onomatopoeia. Socrates dismisses the latter notion, concluding that the proper origins of knowledge are to be found not in the formal characteristics of words themselves but rather in the ideas they signify. The word 'horse,' for example, does not signify a particular object because of any necessary relationship between its arrangement of letters or the sound produced when spoken aloud, but rather because it calls to mind a particular idea which directly corresponds to the animal.[4]

Like Plato, Aristotle was interested in language primarily as the vehicle for thought, and viewed mental states as symbols to which words refer. For both Plato and Aristotle, the conventional aspect of particular languages was not important; rather, the intriguing question was the epistemic basis for connecting words with things, a link which both authors concluded depended upon the reliability of ideas to simultaneously act as referents of words and to signify real objects. Aristotle sums up this chain of signification in *On Interpretation*, where he distinguishes further between written and spoken language:

> spoken words are the symbols of mental experience and written words are the symbols of spoken words . . . The mental symbols, which these [words] directly symbolize, are the same for all, as also are those things of which our experiences are the images.[5]

In claiming that the "mental symbols" that words signify "are the same for all," Aristotle distinguishes between the conventional aspect of 'a language' and the universal function of language in general by conceding that while individual words may be employed conventionally, the mental images they signify are not. Because mental pictures are derived from the experience of "things," and because we all have the same mental pictures of things, Aristotle suggests there must be either an inherent correspondence between all minds and the world, or else all minds must contain an innate set of categories shared by everyone. Aristotle also maintains that while one function of words and language is to facilitate communication, logical determinations (such as causal relationships) are not based in language but rather are merely reflected in it knowledge, in other words, is not 'in language,' but rather 'in the mind.'[6] As he states in the *Posterior Analytics*:

> all syllogism, and therefore all *a fortiori* demonstration, is addressed not to the spoken word, but to the discourse within the soul, and though we can always raise objections to the spoken word, to the inward discourse we cannot always object.[7]

This kind of realism about the connection between language, ideas, and the world was the standard position taken in antiquity, and it informed most of the linguistic theories of the Middle Ages and the Renaissance. Other ancient commentators, such as the Stoics Zeno and Diogenes Laertius, believed that the mind was formed to reflect the rational order in nature, which privileged mental imagery in much the same way as Plato's doctrine of forms. It should be noted, however, that at least one significant strain of Greek philosophy rejected this position in favor of a more rigorous conventionalism: Aristotle reports that Democritus advocated a conventional theory of language, and Epicurus's *Letter to Herodotus* promotes a conventional attitude towards language coupled with a quasi mechanical theory of sense perception and idea formation that is explicitly anti mentalistic.[8] These examples become significant when we see later that the seventeenth century nominalist movement was partially inspired by these classical atomist authors.[9]

On the subject of mathematics as a mode of representation, Plato and Aristotle promote views consistent with their linguistic theories. Plato is famous for his declaration that all students in his Academy should be versed in geometry, but there is little evidence that he himself was an accomplished practicing mathematician. His ontology, however, is explicitly geometrical, and his dialogue *Timaeus* is probably the most frequently cited source for the belief that the universe is a divine geometrical construction. Plato states here that God "judged that order was in every way better" than disorder and established a structure based on the perfection of the regular geometrical solids.[10] This geometrical cosmology is backed by an assumption that mathematics, being a science of the 'eternal and unchangeable,' represents the most secure knowledge and is the highest standard of truth, and Plato's epistemology consequently prioritizes reason above the senses.[11] This doctrine relates to his theory of forms, which posits that our knowledge of mathematics is underwritten by the intellect's perception of archetypal figures that are eternal and exist outside of the realm of the senses. As a general epistemological program, this extreme idealism had only limited influence on later philosophical movements. Aristotle rejected his teacher's doctrine of forms, and while it enjoyed a revival among fifteenth and sixteenth century Neoplatonists and seventeenth century Platonists, Platonic idealism did not agree particularly well with the experimental spirit of the Scientific Revolution. Specifically as a philosophy of mathematics, however, Plato's arguments were enormously influential: Kepler, Galileo, Descartes, Leibniz and many other founders of the 'modern' mathematical method could variously be described as having Platonic conceptions of mathematical concepts and entities, and in the twentieth century there are yet mathematicians who would willingly call themselves 'Platonists.'

Plato's philosophy of mathematics is fully integrated into his general theory of knowledge, and in a number of dialogues he explicitly uses mathematical examples to illustrate his epistemological beliefs.[12] Plato was a metaphysical realist, and held that when we perceive individual objects, we are accessing not their particular qualities but rather their essential nature or the real universal category they fall into. To illustrate this, Socrates presses Theaetetus to describe how universal notions are formed from particular objects, and particularly wonders "through what bodily organ the soul perceives odd and even numbers and other arithmetical conceptions." Theaetetus responds to Socrates' approval that his "only notion is, that these [mathematical objects], unlike objects of sense, have no separate organ, but that the mind, by a power of her own, contemplates the universals in all things."[13] This aspect of Plato's epistemology relates directly to his theory of 'recollection' or reminiscence, which is developed primarily in *Meno* and *Phaedo*, and becomes a paradigm for human apprehension of geometrical ideas. In order to demonstrate to Meno that we are born with the knowledge of essences built into our minds, Socrates questions an unschooled servant about the construction of a geometrical figure. Although he has never learned geometry, the servant is able to answer Socrates' questions, which causes Socrates to conclude that "he who does not know may still have true notions of what he does not know." Socrates explains that the servant is able to 'recover' knowledge of "geometry and every other branch of knowledge," since "there have always been true insights in him, both at the time when he was not a man, which only need to be awakened into knowledge by putting questions to him."[14]

Socrates uses the discussion of recollection as proof of the immortality of the soul, since the objects of truth, which are immaterial and eternal, can only be apprehended by something that is also eternal. It is from this notion that Plato develops his strong dichotomy between sensory (corrupt) experience and rational (perfect) knowledge, and while he applies this model to many kinds of concepts 'truth,' 'beauty,' 'the good' it is particularly well suited to mathematics. In Plato's account, mathematical objects are real objects in the purest sense: they are totally disembodied from physical form, and their existence is not inferred or abstracted from crude matter, but rather is present to the intellect prior to any experience of physical nature. Because of this Plato is ambivalent about the use of language the use of particular words to signify concepts in a particular language, that is but not about the ability of the mental images signified by words to accurately represent true objects of knowledge. In a sense, an idea of an essential nature actually *is* that object, since it is perceived by the soul, which is made of the same 'substance' as the ethereal substratum occupied by the forms.

This is one major difference between Plato's and Aristotle's philosophies of mathematics: Aristotle's approach is integrated into his overall logical method, and therefore is developed more distinctly and explicitly than Plato's. As Deborah Mondiak notes, Aristotle's philosophy of linguistic meaning (as expressed in *De Interpretatione*) informs and is supported by his more epistemological works (such as *Posterior Analytics*, *Physics*, and *Metaphysics*), and she argues that "Aristotle's intent [in *De Interpretatione*] . . . is to give an account of language and its relation to the world that supports, *inter alia*, the realist epistemology of the *Posterior Analytics*."[15] Like Plato, Aristotle was also knowledgeable about the mathematical developments of his day, and he followed Plato in often using mathematical examples to illustrate metaphysical and logical arguments.[16] In a sense, Aristotle's metaphysics is reducible to logic: it is concerned with categories of existence and with the classification of objects according to these categories. Unlike Plato, Aristotle denies primary existence to abstract entities, which is part of his rejection of Plato's doctrine of forms.[17]

One consequence of this position is that metaphysical questions concerning existence require logical definitions. Since Aristotle's metaphysics holds logical consistency as its highest criterion for truth, one might wonder whether Aristotle's notion of existence is simply a logical one.[18] In the *Posterior Analytics*, Aristotle addresses the problem of definitions, noting that a definition cannot confer knowledge of the essence of an object. Using the example of the notion 'triangle,' Aristotle explains that neither the word nor its associated definition reveals the 'nature' of the geometrical object. Aristotle points out that while a definition may be true in the sense that it does not violate any logical criterion for existence, this 'truth' does not prove that the object defined necessarily must exist, and he concludes that "definition neither demonstrates nor proves anything, and that knowledge of essential nature is not to be obtained either by definition or by demonstration."[19] This does not, however, mean that Aristotle believes that essences either do not exist or are not accessible by reason. Aristotle defines "essential nature" instead as "the cause of a thing's existence," and he notes that a syllogistic demonstration (as opposed merely to a definition) which logically deduces the cause of a thing is capable of revealing the thing's essence.[20]

The question then is whether mathematics, which, according to Aristotle, is a logical science, has as its objects real objects whose essences can be causally known, or rather is merely a set of definitions. This discussion takes place primarily in the *Metaphysics*, and the conclusions are somewhat equivocal. Aristotle does state categorically that mathematical objects do have some kind of existence, but suggests that the interesting question is "not whether they exist but how they exist."[21] He begins by surveying the popular opinions on this subject, going into a lengthy and critical discussion of Plato's belief that "the Ideas are substances," which he eventually dismisses. A crucial question for Aristotle is whether mathematical objects exist as 'primary' or 'secondary' substances that is, whether they exist as separate entities or rather as accidents tied to perceptible objects. This discussion has particular relevance to arithmetic, since a central question concerns the status of *number* or "discrete quantity," as opposed to continuous geometrical quantity. Aristotle concludes that, since every instance of discrete quantity is related to a perception of some other object 'three horses,' 'five stones,' etc. numbers are accidents, existing only in relation to some physical substance. He bases this reasoning on the principle that two primary objects cannot occupy the same space; supposing numbers to exist as separate entities would necessitate the simultaneous existence of both the physical object ('stone') and its quantity ('one'), which is a physical impossibility.[22]

Granting mathematical objects existence as accidents, however, circumvents this physical difficulty, since it allows for quantity to be 'in' physical objects without contradiction, and still preserves mathematical objects as 'abstract entities' with a kind of real existence. Aristotle concludes

> Thus since it is true to say without qualification that not only things which are separable but also things which are inseparable exist, it is true also to say without qualification that the objects of mathematics exist, and with the character ascribed to them by mathematicians.[23]

Elsewhere Aristotle argues for the primacy of arithmetic over geometry, since his interest is in explaining the nature of discrete physical quantity, and not quantity in the abstract.[24] Aristotle's preference for arithmetic seems natural given his rejection of Platonic realism: he believes that quantity is 'real' in the same sense that abstract accidents like 'beauty' or 'justice' are, but does not give them primary or prior existence in the manner of Plato's forms. Similarly, Aristotle regards linguistic universals or species as 'real' in that the mental states they represent reflect the actually existing order of nature, but he denies that they are separate objects or archetypes.[25]

The legacy of Aristotle's metaphysics is, of course, far reaching and complex (much more so than I am able to present here), but a general conclusion may be drawn which bears on the rest of this study. Aristotle's ontology of mathematics is tied to his ontology of accident, which places his philosophy of mathematics (and representation more generally) somewhere in between Platonism and empiricism. Over the course of the next two millennia, there are three major positions that recur in the philosophy of mathematics and language: The first holds that mathematical objects and abstract entities are primary substances, which describes Plato's doctrine of forms or the more extreme mathematical realism of the Pythagoreans. The second is that linguistic and mathematical objects exist, but as accidents of physical objects, which is Aristotle's position. The third choice, which was introduced by the skeptical schools of late antiquity and revived in the seventeenth century, is that mathematical objects and abstract entities are neither primary substances nor accidents. Given the fact that the mechanical philosophers of the Scientific Revolution rejected Aristotle's physical ontology, it is not surprising that they also rejected his 'compromise' on mathematical entities. This left only the first and third options: the former, 'Platonic' view was taken up by Galileo and Descartes, while the latter appealed to the nominalists, who argued that abstract entities fall into a certain class of fictional entity 'mental constructs' which have no real existence.

The period directly following the lives of Plato and Aristotle was one of the most important historical eras in mathematics and mathematical astronomy. Euclid's synthesis of deductive geometrical method, Archimedes' studies in mechanical geometry, Appolonius' work on conic sections, and Hipparchus' geometrical astronomy all had great influence on subsequent authors. From a philosophical standpoint, however, of greater interest was the interpretation of these more technical works in later antiquity by members of the Neoplatonic school associated with Proclus (410 85 CE), and the hostile reaction towards mathematics and Aristotelian logic of the Pyrrhonian skeptical movement, which was documented by Sextus Empiricus (fl. 200 CE). It was these Roman era interpreters who, when revived in the late sixteenth and early seventeenth centuries, sparked much of the reaction against scholastic philosophy, and their work therefore provides an important background to the debates that are the subject of the next chapters and deserves summary here.

The most prominent source for later mathematical Neoplatonism was Proclus, the fifth century Byzantine Neoplatonic author who was widely read and admired in the Middle Ages and the Renaissance. There is evidence that Aquinas drew on several of Proclus' writings, which became widely available starting in the fifteenth century and went on to influence Nicholas of Cusa, Johannes Kepler, Ralph Cudworth and Henry More, and even Benedict Spinoza.[26] Proclus' *Commentary on the First Book of Euclid's Elements* is his most explicitly mathematical work, and it provides a Neoplatonic reading of Euclid's geometry that fuses Plato's geometrical idealism with Euclid's deductive method. This work is also significant because it stands, in Glenn R. Morrow's words, as "the only systematic treatise that has come down to us from antiquity" that deals with questions about "the nature of the objects of mathematical inquiry and the character and validity of the procedures used by mathematicians in handling these objects."[27] The *Commentary* consists of a detailed summary and analysis of the definitions and propositions in the first book of Euclid's *Elements*, but most interesting is Proclus' "Prologue," which deals with the metaphysical and ontological status of mathematics and mathematical objects. Proclus applies a generally Platonic epistemology and ontology to Euclid's method, but his analysis of the philosophical issues is, as Morrow notes, more systematic than Plato's own.

In this account, Proclus defines mathematical objects as substances whose existence is situated between objects of sense and those of pure intellect. He ranks mathematical objects above the objects of sense perception, which "fall short of the primary realities in every respect," and allows that while mathematical objects are not Forms themselves, "they stand in the vestibule of the primary forms."[28] He attributes this lower status of mathematical objects to the fact that they are "images" of intelligible reality, rather than direct objects of reality itself, since they are formed in the human mind. Elsewhere he calls them "understandables," by which he means they are objects captured by the "organ" of understanding, as 'sensibles' are objects captured by the senses. Proclus thus views mathematical objects as reflections of the world of the Forms, translated and represented as images in the human mind. Appealing both to Plato's doctrine of recollection and his view of the mind as the 'organ' of the intellect (just as the eyes are the organ of sight), Proclus incorporates mathematics into a consistently Platonic theory of representation: mathematical objects are 'mental images,' but as mental images they directly reflect (with one to one correspondence) the objects they represent, and being drawn directly from pure objects of the intellect, mathematical objects are certain and reliable representations of 'pure' reality.[29]

As mentioned earlier, Proclus' commentary had wide readership throughout the millennium following his death, and his metaphysical commentary, appended as the prologue to his more technical discussion of Euclid, was available to any students of mathematics who turned to the *Commentary* simply as a mathematical textbook. Not all commentators on the sciences in later antiquity, however, saw the appeal of mathematics or of Platonic and Aristotelian philosophy, and the views that rose out of the skeptical schools of this period also had great influence on later natural philosophy. The two most significant schools of classical skepticism were the 'Academic' movement and the 'Pyrrhonian' school.[30] The more significant for our purposes was the Pyrrhonian school, which had its origins in the teachings of Pyrrho of Ellis (360 275 BCE) and was primarily available to later audiences in the writings of Sextus Empiricus, chiefly in the works *Against the Mathematicians* and *Outlines of Skepticism*. Richard Popkin argues that Sextus' works reached a wide audience during the sixteenth and seventeenth centuries, and were quite influential in sparking the skeptical debates that fueled much of early modern scientific inquiry.[31] It is fair to say that seventeenth century nominalist epistemology was strongly influenced by Pyrrhonian skepticism: Gassendi remarks that he was much taken with Sextus in his youth, and while he claims to have ultimately rejected Pyrrhonism, his later writings make much use of the language and argument of that school, and effectively interpret Pyrrhonism for a contemporary audience.[32]

Pyrrhonian skepticism has a kind of systematic ambivalence at its core: it asserts that since we can make no certain claims about knowledge, we must consider both sides of any argument equally and suspend final judgement about the ultimate truth of any claim. Importantly, however, Pyrrhonian skepticism is not necessarily a negative philosophy, since it does not argue that the study of nature or the pursuit of knowledge is futile, but only that we must limit the truth claims we draw from our investigations. The true skeptic has an open mind and is always inquiring, and refuses ever to be satisfied that the essential nature of any object of knowledge is fully known.[33] Pyrrhonian relativism is especially notable because it locates uncertainty in the mechanisms of cognition that perceive and represent reality: the senses, language, and the logical methods of proof and demonstration.

In applying this reasoning to mathematical objects, Sextus attacks the Pythagorean Platonic conception that numbers exist apart from physical things which are numbered. Here his arguments are reminiscent of Aristotle's: Sextus challenges the logic that would allow 'numbers' to be independently real objects, and to reside at the same time within other independent objects. How can two objects exist in the same place, he wonders, and what differentiates the 'number' from the 'numbered object' itself? Sextus shows by example the potential absurdity in Pythagorean mathematical realism: "if numbered objects are {also} numbers," he proposes, "then since men and cows (say) and horses are numbered, numbers will be men and cows and horses and white and black and bearded, if the objects measured should happen to be so." The conclusion he draws is that "numbers are nothing outside and apart from the numbered objects," and he remarks that since "numbers neither exist in their own right . . . nor are the numbered things themselves . . . then we should say that numbers do not exist."[34] Sextus regards the ontological claims made by other philosophers on behalf of mathematics as "superfluities peddled by the Dogmatists," and claims that his destruction of the concept of number "will suffice for the part of philosophy they call physics."[35] This is a much more extreme attitude than the one taken by Gassendi and the later nominalists. Sextus appears to dismiss all of mathematical physics, which Gassendi and the others would not do, but the philosophy of mathematics advocated by Gassendi did (as we shall see) present mathematical objects as 'fictitious' or 'artificial,' and it labeled the deductive and syllogistic methods of Aristotelians and later rationalists empty of real content.

Seventeenth century nominalists were anxious to preserve some criteria for scientific judgement and therefore did not entirely dismiss deductive reasoning or mathematical method; knowing that Gassendi and others read and appreciated Sextus, however, suggests they inherited certain tactics from him for attacking Aristotelian logic and rationalist epistemology. I would not describe Sextus as a 'nominalist,' but his arguments concerning signs and categories are well suited to nominalist interpretation, and it is interesting that the 'discovery' of Sextus' work in Latin Europe coincides with the sixteenth century revival of the medieval realist/nominalist debates.[36]

It is to those medieval debates that I will now turn, where the origins particularly of the linguistic component of the seventeenth century nominalist position are found.

Nominalism and Constructivism in the Middle Ages

The roots of the conflict concerning logic, mathematics, and language in seventeenth century natural philosophy were crystallized in the 'scholastic' philosophy of the Middle Ages, which established not only many of the key categories and arguments for later discussions, but also provided much of the institutional structure and rhetorical convention for early modern debate. The scholastic authors of the thirteenth and fourteenth centuries in particular helped set the parameters for the later debate by explicitly linking ontological questions about the nature of external reality with epistemological issues concerning the proper methods for language and logic (and mathematics), which created a vocabulary for sixteenth and seventeenth century natural philosophers.[37] It was the philosophical legacy left by authors such as Aquinas, William of Ockham, Duns Scotus, and others that, primarily in the context of reconciling Aristotle with Christian theology, expressed the conflict over representation in the classical philosophies of Plato, Aristotle, and the skeptics as a set of core issues: the debate between realism and nominalism.

At its heart, the realism/nominalism debate asks a fairly simple question: do we gain knowledge of nature through perception of really existing and timeless, universal categories of things ('real essences'), or are we more limited in our cognition to apprehension of singular qualities drawn from individual experiences? Extreme realism draws on a Platonic ontology and posits that the mind directly apprehends essences in nature, and suggests a one to one correspondence between concepts (and representations) and the 'real' natures of objects of knowledge. Nominalism, at the other extreme, generally attributes less certainty to knowledge claims, and assumes that we have access only to the singular properties of objects; as a result, our concepts are not real universals, and the representations we form from them are 'names' (*nomen*) only.

If, properly speaking, the philosophy of mathematics does not materialize as a distinct area of scholarship until the seventeenth century, the philosophy of language may be said to have its formal roots in the Middle Ages. Beginning with Augustine, medieval theologians were particularly concerned with the status of 'signs' and their signification; this is not surprising considering that Catholic doctrine was centered around crucial distinctions in interpretation of images and words of vital theological significance. Since the scholastic approach was centered on the interpretation of scripture, and scripture was thought to be an expression in human language of the timeless word of God, it made sense to philosophers to investigate the basis for the human knowledge of God's divine word. This meant exploring not only the nature and limits of human linguistic expression, but also the wider 'language' of signs: 'natural signs' drawn from objects in nature were contrasted with 'conventional signs' of human origin, and questions were raised about the status of human cognition, which dealt in entities that are ambiguously positioned between signs and natural objects. Such study also necessitated investigations of the structural aspects of logic and grammar, which provided the rules for linguistic expression, and called for improvements in these areas to align language more closely with the divine and natural order. These pursuits had prior foundation in classical philosophy, but by the High Middle Ages the linguistic metaphysical approach to representation became one of the dominant areas for philosophical dispute.

Serious consideration of the problem of linguistic representation coincides with the rational turn in theology that fused Aristotle's philosophy with Christian theology and produced the scholastic movement. By the thirteenth century, nearly all of Aristotle's works were available in good Latin translations, and much interest and scrutiny was focused on developing 'accurate' interpretations of Aristotelian philosophy to complement Christian doctrine. This resulted in a shift away from the formerly popular Platonist metaphysics of Augustine towards a more systematically logical approach, and it also brought the study of language and grammar closer to the center of scholastic philosophy. The key figure in this transformation is, of course, Thomas Aquinas. While he is justly remembered for his contributions to rational theology, Aquinas also inherited Aristotle's 'empirical' approach to natural philosophy, which means in practice that he has a greater appreciation for the role the senses play in the acquisition of knowledge than did, for example, Augustine, and Aquinas' account of the mechanism of cognition grants less priority to the 'formal' aspects of things.[38] But it should be stressed that Aquinas fits squarely in the 'realist' tradition: as Scott MacDonald explains, Aquinas' epistemology rests on a thoroughgoing "metaphysical realism," and "he holds that there are real natures of naturally occurring substances and accidents and that these real natures can provide the content for universal categorical propositions."[39] The result is a philosophy of representation that is truly Aristotelian in spirit: less idealistic than Platonism, Aquinas' philosophy of mind nonetheless grants one to one correspondence between human conceptions and language, and grants mathematics a privileged ontological and epistemological status.[40]

While Aquinas certainly may be said to have promoted a consistently realist position toward knowledge of universals, it should be noted that his turn away from the Platonic ontology of Augustine also represents a shift in epistemology that, in the fourteenth century, would produce the more extreme position of nominalism. Meyrick H. Carre contends that Aquinas "stands midway between extreme Realism and Nominalism," in that Aquinas' account of cognition draws on empirical as well as rationalistic methods for the formation of ideas.[41] There was a tradition, however, extending before Aquinas' time to Peter Abelard (1079 1142) which challenged the notion that ideas corresponded directly with 'things,' and questioned whether universals signified anything other than names applied conventionally in language.[42] Abelard's positions are certainly compatible with those of the later nominalists, but nominalism as a 'movement' did not really take off until the writings of William of Ockham became widely known in the mid fourteenth century.

Ockham's position was part of a wider theological debate at the time concerning the status of universals that touched on such sticky theological issues as the identity of the trinity and the metaphysics of transubstantiation. A number of theologians, including Scotus, Walter Burley, and John Wyclif offered solutions to the 'problem of universals' that were moderately realistic but which also attempted to incorporate elements of the nominalist critique of extreme Platonic realism. Scotus, in particular, suggested a model in which objects might have both formal (universal) existence in the mind and real (particular) existence in nature as a kind of compromise between realism and nominalism.[43] While this view attracted some, it did not win over Ockham, who denied the real existence of any universal natures at all. Ockham's philosophy influenced a further generation of nominalists, which included John Buridan, Nicole Oresme, and, perhaps most stridently, Nicholas of Autrecourt, who rejected not only universals but also nearly all of Aristotle's philosophy.[44] Nonetheless, Ockham's philosophy stands as the most thoroughgoing nominalist system of the Middle Ages, and most closely anticipates the views of seventeenth century philosophers such as Gassendi, Hobbes, and even Berkeley.[45]

At its heart, Ockham's position is notable because it combines two arguments that will form the heart of the seventeenth century nominalist critique I will describe in later chapters: first, it denies the existence of real universals and provides an account of representation that focuses on the purely conventional aspects of signification, and, second, it presents a critique of Aristotelian ontology and formal categories of existence.[46] The central issue in the debate over the nature of representation is whether 'mental images' (which nearly everyone acknowledged existed) have a one to one correspondence with the natural objects they signify, and whether that correspondence (one to one or otherwise) captures some essential or universal nature of the objects perceived. Aristotle recognizes that experiences are of particular objects, but grants nonetheless that the mind is able to apprehend the universal essence or category that an object falls into, and thus what is represented in the mind is not just an image of the object in its particularity, but also a reflection of its universality. Ockham, on the other hand, denies that concepts correspond to things at all. His reasons for rejecting the concept thing equation are somewhat complex, but essentially boil down to his denial that real universals exist in nature. Ockham's logic is as follows: words signify concepts, and concepts signify things. Signification is a relation, and relation is always of the 'one to the many' (i.e. how an individual may be categorized as part of a group), so concepts are universals. But every real 'thing' in nature is an individual, so concepts cannot be things; hence, concepts exist only in the mind and have no necessary correspondence to objects in nature. As Ockham explains:

> properly speaking, no universal belongs to the essence of any substance, for every universal is an intention of the soul or a conventional sign and nothing of either sort can belong to the essence of a substance. Consequently, no genus or any species nor any other universal belongs to the essence of any substance.[47]

Ockham is quite willing to accept that universals exist in the mind—he simply regards them as artificial constructions, and denies they necessarily correspond to physical reality.

Much of Ockham's epistemological stance rests on his linguistic conception of mental processes, in which he centers on the 'propositional' nature of signification. 'Mental terms' are not themselves 'words,' but when composed to form propositions they become "mental words" which "belong to no language" and "reside in the intellect alone."[48] Ockham himself likens this stance to Augustine's position, but whereas 'realist' philosophers of language use the similarity between mental images and written or spoken words to justify claims about the certainty of language, Ockham does the opposite: by concluding that signification is an attribute of propositions and that words are essentially propositions, he argues that words signify only by virtue of the propositional relation between concepts and things, and not because of an essential correspondence. Propositions, Ockham explains:

> occur only in the mind, in speech, or in writing; therefore, their parts can exist only in the mind, speech, and in writing. Particular substances, however, cannot themselves exist in the mind, in speech, or in writing. Thus, no proposition can be composed of particular substances. Propositions are, however, composed of universals; therefore, universals cannot conceivably be substances.[49]

What is remarkable about this argument is that Ockham manages to tie an epistemological claim about the nature of language together so tightly with an ontological assertion about the nature of substance; this gives Ockham's philosophy its 'thoroughgoing' nominalism. On Ockham's view, propositions signify nothing other than their logical truth value, which reduces the prospects for having certain knowledge of the natural world to a semantic operation; as Claude Panaccio notes, "Ockham refuses to admit special objects in the world as significates for propositions," meaning that "truth and falsity . . . can be accounted for without enriching ontology further."[50]

Ockham's theory implies that while we experience only singular objects, when we speak (write, think, etc.) we speak of universals; thus, there is a formal 'disconnect' between language and reality. This is not, however, to say that the native experiences used to form propositions in the mind have no relation to nature; on the contrary, we perceive individuals with accuracy, and it is only in the process of concept formation that experience is universalized as propositions. This preserves a measure of 'reality' in language and knowledge, since propositions are constructed out of this genuine perception, but it does introduce a notion of 'artificiality' to knowledge as well, since it acknowledges the constructedness of language.

Ockham's formal approach to language is reflected also in his view of mathematics, which André Goddu calls a "pragmatic conception . . . that makes mathematics into a language or another tool of analysis." Ockham does not write much about mathematics, but he is notable for disavowing the mathematical realism of previous scholastic philosophers by moving "away from mathematics as a discussion about objects and entities to mathematics as a language and a formalism that can be interpreted in many ways and applied to all disciplines."[51] This indeed reflects Ockham's epistemology, since mathematics, interpreted as a 'language,' describes quantity and figure universally and thus could not possibly describe objects that have actual existence in nature.

For example, in *Quodlibetal Questions*, Ockham asks whether "it can be proved evidently that a quantity is an absolute thing distinct from substance and quality?"[52] In other words, he inquires into the independent ontological status of number. In general, Ockham's response is that there is no evidence, either logical or empirical, that quantity can be separated from the substance in which it inheres. He notes that in defining quantity as an 'accident' one must tread carefully, since (following Anselm) accident can be defined more or less broadly.[53] Ockham argues that quantity qualifies as an accident "since 'quantum' and 'quantity' are predicated contingently of a substance solely because of a local change in that substance."[54] This does not, however, mean that quantity is distinct from substance or could be separated from it. Ockham writes that the statement "'every quantity is an accident' is true" by virtue of the fact that "in that case 'quantity' is being taken for a spoken word or a concept."[55] In other words, quantity has as much external reality as a word or an idea that is to say, *mental* existence but not physical existence. While he does not develop a theory of quantity or number further, I think examples such as this suggest his view of mathematics was consistent with his nominalistic approach to language and concepts.

More significant than Ockham's treatment of number and mathematics he was, after all, not particularly interested in those subjects is his position concerning the 'constructedness' of universal terms. Here his nominalist ontology coincides with theological voluntarism: one of the points Ockham wants to make is that God has the absolute power to create any order of nature that He wishes, meaning that Ockham rejects any necessary logical or physical relations between singulars. For Ockham, what *is* and what *is possible* are two separate issues. God's *potentia absoluta* allows the creation of alternate worlds in which the current order of nature does not apply. Our world is utterly contingent on God's will, which is not bound by even logical necessity.[56] Counterfactual (or 'incommensurable') physical conditions are not impossible in any necessary sense.

For example, in his commentary on the *Sentences* of Peter Lombard, Ockham moves from the nominalist assertion that a universal "has only a logical being in the soul and is a sort of fiction" to a discussion of the ontological status of other kinds of 'fictions,' such as chimeras and centaurs.[57] As Ockham points out, fictions are not ontologically real because God has not willed them so, but they are nonetheless "from God" in the sense that "it is possible to have intuitive knowledge, both sensitive and intellectual, of a thing which does not exist," and it is impossible for us to have knowledge of things that God could not create if He so wished. Likewise, we can understand things that have ceased to exist by the power of God (for example, a star that has been destroyed) because God preserves the intuitive knowledge even after the object itself has been removed.[58] As Ockham clarifies in *Quodlibetal Questions*, "figments are from God . . . Just as falsehoods are from God because they are real things," since "there are no such objective beings that neither are nor can be real beings."[59] Ockham engages here in a lengthy discussion of whether God can create logical contradictions; while he concludes that God cannot cause something to simultaneously be and not be, it is certainly within His power to change states in successive instants, as in removing the Virgin Mary's state of original sin, or making a black thing white.[60] For this reason God is not bound by future conditionals, since He has "an intuitive cognition of all things which can or cannot be made," as well as the ability to change them at any time.[61]

Despite his fairly radical program of nominalism, however, Ockham at least maintains the impression that he is not contradicting Aristotle. Even while he pulls apart the logical relations in the Aristotelian theory of signification, Ockham nonetheless preserves much of the original vocabulary (such as Aristotle's distinction between types of discourse), and he makes at least a half hearted attempt to apologize for Aristotle's position on universals, claiming "all authorities who say that universals belong to the essence of substance . . . should be interpreted as saying only that universals indicate, express, designate, and signify the essences of things."[62] This shows that, on the surface, Ockham presented himself more as a reformer than a revolutionary, which was probably a judicious position to take in light of church politics at the time. Ockham's follower Nicholas of Autrecourt, on the other hand, adopted a decidedly more antagonistic stance. In his *Universal Treatise* (*c.* 1340), Autrecourt proposed a vigorously nominalistic approach to universals, coupled with a scathing attack on Aristotle, concluding "that the assertions of Aristotle in various places are false, and sometimes in certain places there is only fiction."[63] Autrecourt's treatment of the problem of universals is similar to Ockham's (though less clearly presented), but it is noteworthy because he pushes the skeptical subtext of nominalism further than Ockham, and because he uses the nominalist epistemology to introduce an ontology of atomism.

Like Ockham, Autrecourt reduces knowledge and understanding to a mechanism of 'mental propositions', whose "proper concepts" (i.e. the terms in the propositions) are provided by sense experiences of individual objects.[64] The senses are what make our propositions commensurate with external reality, since Autrecourt denies that concepts without empirical basis can be established as 'facts.' As an example he presents the case of a blind man, whose logical statements about the nature of color may be judged 'true' (as propositions), but who ultimately does not have 'knowledge' of color because his mind lacks concrete references to actual experience.[65] This case well summarizes Autrecourt's epistemology: mere logical correctness does not equal demonstrative knowledge, since knowledge is founded only on the experience of particulars. The blind man is a metaphor for human inquiry in general: "the intellect is certain concerning everything which is evident to it," but such evidence comes "only according to an act of the senses." Thus we may have a limited kind of certainty about propositions that involve concepts "properly" obtained from individual experience, but for all arguments that concern universals our propositions are as logical and empty as the blind man's concerning color.[66] This approach underlines the potential skepticism inherent in nominalist metaphysics a skepticism Ockham worked hard to avoid and suggests that probability, rather than certainty, is the best that can be hoped for in knowledge of natural phenomena.[67]

Without venturing further into the details of Autrecourt's epistemology, there is one other remarkable feature of his philosophy, and that is his atomism. I note this because, as will be seen in the next chapter, atomist ontology was frequently associated with nominalist epistemology, and in the case of Gassendi the two were intertwined as the foundation of his system of natural philosophy. Goddu notes that Autrecourt was the only medieval nominalist to present an atomist ontology, but points out that there is evidence that later authors including Kenelm Digby and Hobbes may have interpreted Ockham to provide evidence in support of such a position.[68] Autrecourt's arguments on this subject are presented as responses to Aristotle's famous critique of Democritean atomism in the *Physics* and elsewhere, and center on Aristotle's refutation of Zeno's paradoxes concerning the divisibility of motion. These paradoxes the 'stadium' and 'Achilles' in particular point to the problems that arise if one considers the motion of an object across successively halved distances during successively halved times: the result is that, as one approaches infinitely small increments, motion appears to cease, and in another instance a 'faster' object will eventually be overtaken by a 'slower' one.

One possible solution to all these paradoxes is to deny that infinitely small increments actually exist beyond a certain ultimate minimum an ‘indivisible unit’ or atom which is what Democritus and Epicurus argued (and used as an argument in favor of atomism), and Autrecourt follows this argument. Suppose, he offers, “the fastest movable thing moves in three instants through a space *abc* composed of three indivisibles,” while an object half as fast moves through the space in twice the time. Because the space is composed of three indivisibles, and because time is imagined to be composed of ‘instants’ which are also indivisible, it is impossible to move less than one unit of space in any given moment, or faster than one instant through any given space. What maintains the relative speeds of the objects to one another, Autrecourt proposes, is not a continuous velocity, but rather a kind of discontinuous succession of motions across successive instants of time:

> If a movable object twice as slow moves through that space [*abc*], I say that it will reach *a* in one instant and rest for another instant; similarly it will reach *b* in one instant and rest at it for another instant; similarly with *c*.[69]

In this rather unintuitive and wholly non Aristotelian physics of movement, motion is actually a succession of (assumedly unperceivable) fits and starts, and the atomistic nature of both space and time explains differences in velocity.

I will not take the time to analyze Autrecourt’s proposal in detail or to compare it to similar arguments by other authors, but I will note that it is exactly the kind of strategy (if a somewhat awkward example) used later by many of the ‘empiricist’ nominalists/atomists of the seventeenth century. Classical atomism is predicated on the assumption that experience should outweigh logic when there is a dispute, and that the simplest and most pragmatic answer should be sought before the more complicated and speculative. Autrecourt’s argument does not explain what mechanism allows an object to ‘rest’ before continuing on its path, but nor does it advance a complicated theoretical model which it claims gives a thorough causal explanation of motion as a general phenomenon (like Aristotle’s natural motions or the highly problematic antiperistasis). It simply remains agnostic on ultimate cause, while presenting the ‘simplest’ physical account. This is certainly in the spirit of Ockham (remembering his ‘razor’) as well as the empiricist and pragmatic Epicurus, and it suggests that nominalism and atomism share a basis in epistemology that favors the senses and clear physical description over excessively complicated rational deduction.

As the medieval nominalists show, precursors to many of the arguments central to the seventeenth century debate about the proper methods for natural philosophy can be identified not only in classical philosophy, but also in the output of the medieval scholastic authors. Medieval authors tended to focus more intensively on linguistic and logical questions in epistemology, but as I have suggested their arguments also directly influenced conceptions of mathematical methods and objects. One factor that probably limited mathematical discussion was philosophers' focus on Aristotle between the twelfth and fourteenth centuries; the reintroduction of Plato into European philosophy in the fifteenth century sparked renewed interest in geometry and geometrical cosmology, since Plato's writings took a more speculative interest in geometry than did Aristotle's. This 'rediscovery' of Plato brought with it a greater appreciation of geometry and revitalized geometrical idealism as a mainstream epistemology, but the sixteenth century in particular also saw a significant interest in logic, grammar, and the philosophy of language, much of which was derived from the equally important rediscovery of classical atomism.

Grammatical Criticism and Classical Skepticism in the Sixteenth Century

In the fifteenth century, renewed interest in Plato's philosophy led to what many historians have described as a change in the philosophical and scientific mentality in Western Europe. This shift centered on a critique of Aristotelian philosophy, which in the sixteenth century produced two distinct (but related) philosophical trends that bear strongly on the development of the epistemological and ontological orientation of the seventeenth. The first was the re thinking of medieval scholastic grammar and logic, which increasingly became oriented towards a structural and formal approach to language in contrast to the realist and essentialist Aristotelian method of Aquinas. This approach to epistemology often reflected the concerns of the medieval nominalists, and its chief exponents such as Petrus Ramus and Francisco Sanches were read and commented favorably upon by Gassendi and other seventeenth century nominalists. The second trend is in many ways the opposite of the first: the rehabilitation of Plato inspired greater interest in geometry, and elevated geometry in the eyes of many natural philosophers to a science of the 'language of God.' This somewhat reversed the tendency of medieval practice, which prioritized arithmetic, but was very much in line with a universalist belief in the ontological reality of conceptual and formal entities that is reflected in Aquinas and especially Augustine.

Despite the many reforming efforts of humanist scholars in the fifteenth and sixteenth centuries, the medieval approach to logic inspired by Aquinas and others was not seriously challenged until around 1530.[70] Humanist critiques had tended instead towards grammatical reform, turning back to classical authors in an attempt to recover the 'authentic' style of the ancient rhetoricians. This movement included such notable scholars as Lorenzo Valla, Philip Melanchthon, and Thomas Linacre, and it was out of the humanist concern for proper style and structure in language that philosophical explorations of the nature of language proceeded.[71] Grammatical reform necessitated an examination and critique of prominent Aristotelian concepts in logic and linguistics such as his ten categories and contributed to an explicitly 'structural' approach to language, in which the dependency of language upon formal relationships between parts of speech was highlighted. These structural investigations of grammar in turn sparked scholars' interest in the formal and logical aspects of language, leading, as Timothy Reiss explains, to a distinction by the late sixteenth century "between language as means of rational communication (grammar and rhetoric) and language as means of discovery (logic)."[72] Humanist studies of rhetorical modes such as dialectic also supported this logical orientation by focusing attention on argumentative strategies; Valla, for example, based a study of rhetoric on the analysis of evidence needed to support propositions in disputation.[73] As evidence for this trend towards grammatical and logical analysis of language we need only look to the many authors who published complementary works on both subjects. Petrus Ramus was one such example, producing influential grammar and logic primers in the mid sixteenth century, and his definition of grammar mirrored his definition of logic: grammar was referred to as 'the art of speaking well,' while logic 'the art of *thinking* well,' suggesting that Ramus conceived the two 'arts' to be twin aspects of a single logico linguistic faculty.

Ramus's structural approach to language directly influenced Gassendi, Locke, and other members of the seventeenth century nominalist tradition I will be describing. Ramus was a radical reformer who challenged not only scholastic grammar and logic, but the entire basis of Aristotelian philosophy. He gained notoriety in Paris for several well publicized disputations with university faculty concerning Aristotle's logic, and his writings were quite popular, particularly in England, where translations of his *Rudiments* and *Latin Grammar* were published in 1585, and his *Logic* as early as 1574.[74] In formal respects, Ramus maintained many of the logical conventions of scholastic authors, but in his epistemology he differed significantly from Aquinas and the Thomistic sixteenth century author J. C. Scaliger, denying that concepts expressed in language mirrored a necessary ordering of nature that reflected the 'language of God.' This does not mean that Ramus was a nominalist, but his approach described as "formalist" by G. A. Padley, and logically 'atomistic' by Walter Ong inspired the later nominalist program by highlighting the ability of a formal and structural critique of language to undermine rationalist epistemology and ontology.[75]

Central to the importance of Ramus's formalist logic is its denial of a Platonic basis for the correspondence between ideas and things in the world. Ramus writes that "words are nothing else but notes of matters signified," and he develops a notion that 'ideas' are not mental representations or images of natural objects, but rather are logical 'containers' that function by holding certain significations.[76] Words then signify not images of objects, but 'ideas,' which are mental tokens implying certain significations, and logic involves the manipulation of these tokens to form propositions that may be judged ‘true’ according to formal conventions. Because of this formalistic rendering of ideas, Ramus denies we have the ability to make true causal determinations about natural objects, which challenges not only Plato’s doctrine of forms, but also Aristotle’s rational syllogistic method. A syllogism, according to Ramus, demonstrates only the logical correspondence between mental tokens, and says nothing about the ‘real natures’ of things in the natural world.[77] Ramus also extended this logical formalism to mathematics, which he considered an extremely useful art, but one epistemologically on par with language indeed, Ramus suggests that mathematics *is* in fact a kind of formal language, which makes use of the same logical operations as grammar and logic. In his discussion of mathematics, Ramus dismisses Platonic idealism, claiming that ‘ideas’ are nothing more than logical tokens, or as Nicholas Jardine explains, a “geometrical theorem [is] a generalization about the results obtained in the measurement of real bodies, not a truth about abstract geometrical entities.”[78] Ramus nonetheless maintains that there is *some* connection between ‘real’ objects and mathematical or linguistic ideas, but that this relationship is expressed by a formal mechanism, and not a direct one to one correspondence.

While Ramus was certainly a controversial and influential figure (as much despised by Kepler as he was admired by Gassendi), his method was still firmly rooted within the Aristotelian grammatical tradition, even as it tried to reform that tradition. A far more radical and anti scholastic approach was taken by the Spanish physician, Francisco Sanches (1551 1623), who while not a grammatical author challenged the epistemological foundations of Aristotelian thought on an explicitly skeptical and nominalist basis.[79] Sanches identified himself with the ancient tradition of Academic skepticism that denied the possibility of any demonstrative knowledge, which gives him the title of his major work, *Quod Nihil Scitur* (that nothing is known), published in 1581.[80] It seems evident, however, that Sanches’s position takes little real foundation in Academic skepticism, which was anti empirical, and that he must have been influenced at some point by the writings of Sextus, who made the slogan ‘nothing is known’ famous. In any event, despite maintaining a friendly correspondence with mathematical natural philosophers like Christoph Clavius, Sanches developed a philosophy that denied the existence of real universals in nature and challenged the certainty of logical and mathematical demonstrations. It is Sanches’s philosophy, more than Ramus’s, which may be seen as a direct predecessor of the Gassendist nominalist program, since it fused the epistemological platform of Ockham and the medieval nominalists with the empirical skepticism of Sextus that, in similar combination, would inspire Gassendi in his critique of Cartesian rationalism.

Sanches’ epistemology centers around two apparently contradictory concepts: that “knowledge is perfect understanding of a thing,” and that “nothing is known.”[81] The obvious conclusion is that since nothing can be known, it is impossible to acquire perfect knowledge of a thing, and in one sweep Sanches dismisses the realism of both Plato and Aristotle. Sanches does, however, distinguish between objects in the mind and objects in 'reality,' but he rails against the mistaken belief that "non existent things" such as "Democritus's Atoms, Plato's Ideas, Pythagoras's Numbers, and Aristotle's Universals, Active Intellect, and Intelligences" provide any genuine insight into "unknown truths and the secrets of Nature."[82] Instead Sanches presents a purely linguistic conception of mental objects, claiming "every definition, and almost every inquiry, is about names," and concluding that "we cannot comprehend the natures of things."[83] Sanches contrasts his position with the belief that words convey the natures of objects which opinion he calls "worthless" and proposes instead that words derive signification simply "from the arbitrary decision of him who applies them" and that they have "no power to explain the natures of things."[84] Unfortunately, he offers no logical mechanism to explain the specific functioning of words and concepts, but he does promise to reveal such in a future work (alas never produced).

Sanches's denial of the knowability of real natures and his conviction about the arbitrary and conventional nature of language are rooted in nominalism: he says that 'species' (or universals) "either are nothing, or (failing that) are a kind of 'image making.'" By citing "image making," Sanches apparently means to imply that species have a *kind* of existence, but only as 'images' in the mind, which reflect human convention and not genuine experience. He repeatedly denies that universal notions can be found in nature, and concludes instead that

> Individuals alone exist, and can be perceived; it is only of individuals that knowledge can be possessed, and only from individuals that it can be sought. If this is not so, show me where those "universals" you speak of occur in nature; you will admit that they occur in the particulars themselves. Yet in those particulars I cannot see anything that is "universal"; everything in them is particular.[85]

To this contention he appended the fairly negative observation that since there are an infinity of individuals in nature, there can be no "science" of individuals; given his claim that universals do not exist, this would seem to suggest that no 'science' or knowledge is possible at all. But Sanches is not entirely negative: he notes that we are able to perceive at least the "outward appearance of things" with the senses, and that the mind considers those images, from which it attempts to construct knowledge.[86] The capacity to render knowledge from experience is unfortunately quite limited for "luckless humanity," but through the application of both "experience and judgement" some positive steps might be taken, although it will not be found in "syllogisms and divisions and categories and other mental operations of a similar sort."[87] Rather, by the use of experience and judgement which must be employed together to balance each other's imperfections some tentative steps may be made, and Sanches suggests that although perfect knowledge through experience is impossible, the more empirical evidence one acquires, the better equipped is one's judgement of natural phenomena.

Ultimately, Sanches' epistemology is more pessimistic than Gassendi's and Hobbes's would be, but his work is also dogmatically anti scholastic and in a sense moralistic: his aim is to reproach scholars for the prideful conceit that they hold universal knowledge, and his repeated assertions that we 'know nothing' are in part a rhetorical device designed to hammer this point home. The final dozen pages of the work are devoted to a consideration of how this epistemology may be applied to a series of natural phenomena, medical conditions, and logical dilemmas, and it is evident that Sanches has a real interest in the practice of natural philosophy. Sanches's program of doubt is not a philosophical strategy designed to be whisked away to reveal a glorious path to certainty (as Descartes was to later attempt), but rather an honest and from the tone of the writing painful concern about the limits of scientific knowledge. The conclusion is not to 'give up' because of human limitations, but rather to keep plodding along, with the explicit proviso that we do not assume too much about our mental abilities. In this respect, Sanches's work also prefigures the 'mitigated skepticism' of Gassendi, which sought to put a positive 'spin' on our epistemological limitations. Where Gassendi goes further than Sanches, as we will examine next, is in applying a rigorous program in support of skeptical empiricism, and in attempting to devise an ontological framework supportive of nominalism and an empiricist methodology.

2 Ontology Recapitulates Epistemology: Gassendi, Epicurean Atomism, and Nominalism

Pierre Gassendi has been widely credited with two major contributions to seventeenth century natural philosophy: The first is the revival of the tradition of classical atomism associated with Democritus, Epicurus, and the Roman poet Lucretius, which had wide influence and appeal among the mechanical philosophers.[1] It is clear, for instance, that Robert Boyle was familiar with his works and influenced by his corpuscular philosophy, and there is also evidence that Gassendi's corpuscular system had a significant influence on Newton's early philosophy.[2] The second major contribution was his programmatic promotion of nominalist epistemology and ontology, which reinforced his empirical/mechanical philosophical system. Gassendi has also been noted as a source of the theological voluntarism that was particularly popular in seventeenth century England, and his voluntarist conceptions have a firm basis in his ontology.[3]

While Gassendi was considered by his contemporaries to be among the leading scientific practitioners many considered him Descartes' most legitimate rival he is mostly remembered by historians for his reintroduction of Epicurean atomism. He has certainly not been presented as a philosopher of mathematics of any standing.[4] My purpose here is not to rehabilitate Gassendi's status in this regard, but to highlight his treatment of philosophical issues related to mathematics that are consistent with approaches taken by later figures who most decidedly were mathematicians, such as Barrow and Newton. After all, whatever modern scholars may think of him today, Gassendi had a significant influence on contemporary natural philosophy.[5] Gassendi's own nominalism is inextricably fused with his influential version of the mechanical philosophy, and his general program of epistemological reform, articulated over the course of his lifetime, often centered on mathematical objects and categories.[6] Since many of his positions were presented in his public and widely read commentary on Descartes' *Meditations*, it is reasonable to assert that his views regarding mathematics, as well as his atomism, were known to the general philosophical community.

Natural philosophers of the 1630s and the 1640s would also certainly have been aware of the fairly dramatic feud between Descartes and Gassendi.

While they shared a commitment to a mechanical worldview, in many regards the two philosophers were diametrically opposed. Descartes advocated an ontology in which corpuscles were divided into three major classes large 'globules,' small particles, and subtle luminous matter but he held that corpuscles were theoretically divisible indefinitely. By identifying matter with extension, he maintained that the universe was a plenum: there was no void, and all motion consisted of particles pushing against one another. Atomism, on the other hand, asserts by definition that there is a physical limit to the divisibility of matter. Gassendi (following Epicurus) held that atoms were the least divisible units of matter, and that they interacted freely in a vacuum. These two views may be seen as extremes in a fairly wide spectrum; between their positions were philosophers who either adhered to no explicit corpuscular theory on the basis of agnosticism (Boyle), those who expressed a belief in atoms but outlined no clear ontology (Locke), or those who flirted with several theories before adopting a middle road (Newton).[7]

Although the rift between Gassendi and Descartes has been discussed in recent secondary literature (particularly by historians of philosophy), the Gassendist challenge to Descartes has still not been thoroughly investigated by many scholars.[8] One task of this chapter is to contextualize that split in light of contemporary attitudes towards logic, language, and mathematics, which to seventeenth century practitioners were undeniably seen as related and fundamental aspects of natural philosophy. Despite major re evaluations of the Scientific Revolution, the predominant assumption among historians is still often that Cartesian mathematization and mathematical realism characterize the major advances of the period.[9] However, the nominalist tradition of Gassendist atomism and epistemological skepticism played an important role not only in the development of theories of matter, but also in the mathematico empirical tradition leading to Newton and beyond. Gassendi's ontology cannot be separated from his epistemology; his skepticism and philosophy of representation stemmed directly from his acceptance of Epicurean atomism. This chapter will briefly introduce the basic components of Gassendi's physical ontology, and will then consider Gassendi's theory of representation which includes views on language and mathematics in relation to his ontology. I will also discuss what Lennon has described as "the battle of the Gods and Giants": the war of words between Gassendi and Descartes, with particular emphasis on their opposing attitudes towards language and mathematics. Finally, I will analyze Gassendi's circle and influence, and discuss the scope and relevance of his work in the French and English natural philosophical communities in the mid seventeenth century.

Gassendi and Epicurus: The Physics of Atoms

In many ways, Gassendi stands apart from many of his better remembered colleagues. While he was certainly an avowed practitioner of the 'new science,' some historians have noted that his training and methods seem to point back to an earlier era. In *Gassendi the Atomist*, Lynn Joy argues that what separated Gassendi and Descartes was Gassendi's commitment to humanist historical methods of demonstration and justification. Despite resemblances to certain contemporaries, Joy writes:

> Gassendi was not a philosopher in the same sense as Descartes, or a scientist in the same sense as Newton, or an empiricist in the same sense as Locke. His methods of pursuing scientific and philosophical inquiry were inextricably linked to his genre of the history of philosophy, and this humanist genre actually posed serious challenges to the genres of science and philosophy employed by Descartes, Newton, and Locke.[10]

Gassendi's erudite style, with an indulgence for classical references and ornate Latin sentences, certainly differs superficially from the more colloquial styles of Bacon, Galileo, or Descartes all of whom also wrote at times in the vernacular, while Gassendi employed a notoriously cumbersome humanist neo Latin in all of his published works.

Nonetheless, Gassendi maintained an interest in some of the most pressing technical and scientific questions of his day throughout his lifetime. He corresponded extensively with colleagues concerning problems in astronomy and physics, and his *Opera* contains astronomical texts and biographies of several prominent astronomers written between 1631 and 1655. Gassendi was an accomplished astronomer himself, and he corresponded with Kepler regarding eclipses and the motions of Mars and Venus.[11] As early as 1621 several years before the publication of his first work (an attack on Aristotelian logic) he described his own careful observation of an eclipse in a letter to a patron, Henri du Faur de Pibrac, whom he enlisted for assistance in getting a missive to Kepler.[12] And, in 1642, Gassendi published *De motu impresso a motore translato*, in which he was the first to report actual, observational verification of Galileo's famous thought experiment in which balls are dropped from a moving ship, demonstrating the principle of inertia.[13] Clearly, his interest in scientific epistemology was rooted in practical experience with the sciences, which he attempted to unite in his magnum opus, the *Syntagma Philosophicum* of 1658.

Gassendi's critique of Scholastic philosophy extended beyond a reformulation of ontology; he also wanted to reform logic, and to do so he turned to an analysis of the traditional logical methods used in the schools. This meant that Gassendi was by necessity steeped in that tradition, and certainly contributes to Joy's interpretation. Gassendi's style was that of the humanist 'man of letters,' but his philosophy was nonetheless in touch with the major currents of seventeenth century science. Gassendi was concerned with physical questions of matter, space and motion, and with questions about how human beings receive, interpret and represent natural knowledge. His natural philosophy developed out of an animus towards Aristotelian logic and methodology that spanned his entire career, from his early days as a teacher of logic at Aix en Provence to his last appointment, as professor of mathematics at the University of Paris.[14] In his first publication, the 1624 *Exercitationes paradoxicae adversus Aristoteleos*, we get a sense of some of the themes that Gassendi would develop over the next thirty years: most prominently, Pyrrhonian skepticism and a critique of certainty in language and logic.[15] His 1649 treatise *Animadversiones in decimum librum Diogenis Laertii* was his first major exposition of Epicurus' atomist physics, but in the intervening years, Gassendi had developed an overall system that featured nominalism as a central component linking his epistemology to his ontology.[16]

Gassendi relates that he was impressed at a young age by Hellenistic alternatives to Aristotelian philosophy. But Gassendi was not only influenced by Epicurus' system of atoms and void: early in his career he was much taken with the philosophy of Pyrrho of Ellis (*c.* 360 70 BCE) and his much later follower Sextus Empiricus (*c.* second century CE), whose motto 'nothing is known' Gassendi adopted in the *Exercitationes.*[17] In the Preface to that work, Gassendi states:

> of all the opinions, none ever pleased me so much as the {incomprehensibility of things} extolled by the Academics and Pyrrhonists. Indeed, after I had been given to see how great a gulf divides the Spirit of Nature from the human mind, what else could I think but that the inner causes of natural effects totally elude human investigation?[18]

Thus, while Gassendi has been associated with the renewal of atomism, it is perhaps more accurate to say, as Joy maintains, that his thought was an extension of the neo Classical humanist revival of the late Renaissance. There is an undeniable continuity between Gassendi's philosophy and sixteenth , fifteenth , and fourteenth century antecedents, and this fact as much as any other helps explain his ultimate failure to communicate with Descartes.

Although Gassendi became interested in the physical and ethical system of Epicurus fairly early in his intellectual career, it was not until relatively late in life that he actually published on Epicurus' physics.[19] Gassendi's lengthy exposition of Epicurean atomism, the *Animadversiones*, did not appear until 1649, and his re working of that material in the *Syntagma* was published only posthumously in the *Opera* of 1658.[20] A simple glance at the chronology of his works, however, shows that throughout his entire life Gassendi was profoundly concerned with epistemological, ontological, *and* technical questions, all of which he attempted to unite in the *Syntagma*. Gassendi's ontology of Epicurean atomism cannot be separated from his epistemology, which combined Pyrrhonic skepticism, empiricism, linguistic nominalism and (as we will see) mathematical constructivisim. Earlier generations of historians (notably Richard Westfall and Alexandre Koyré) have questioned whether Gassendi's project was a fully realized and systematic natural philosophy.[21] It is certainly a valid question to ask whether Gassendi's work was *received* as a systematic philosophy by his contemporaries, but Gassendi himself undoubtedly intended it as such. I will attempt to draw these various strands together to demonstrate Gassendi's original and important contribution to theories of representation in the emerging mechanical philosophy.[22]

Whereas Descartes claimed to build his mechanical system from the ground up, Gassendi found many of the pieces for his new philosophy in the physics of Epicurus. In the *Syntagma*, Gassendi presented his ontology in detail. Although this work draws heavily on Epicurus' work, the *Syntagma* is not simply an extended commentary on or epitome of Epicurean philosophy, but rather is a synthesis of Gassendi's life's work and unique contribution to philosophy, as Gassendi's modern biographer Howard Jones stresses.[23] Gassendi begins the physical section which runs to 1,277 pages in the *Opera* (around a third of the entire work) by contradicting Aristotle concerning the existence of the vacuum, which Gassendi describes as an absolute space that exists apart from any matter that occupies it. Unlike Epicurus, Gassendi believed that the universe was created by God and is not eternal, but he understood the 'universe' to mean the boundaries set by God to contain matter, and not infinitely extended space. Here Gassendi shows his allegiance to medieval nominalists, writing "space would be boundless before God created the universe . . . [and] would still remain if He should destroy the universe, and that of his own free will God delegated this determined part of space's realm in which he created the universe."[24] Space (vacuum) exists, therefore, but is not properly a substance or an accident and does not fall into ordinary categories of existence. Space is also immobile and entirely unaffected by the movement of matter: "when some object, or part of the universe, moves from its place, the space in which it is situated does not move along with it, but remains motionless as it is left behind."[25] Gassendi's approach here is identical to the stance later taken by Samuel Clarke on behalf of Newton in combating Leibniz. By defining the term 'universe' as a 'boundary' carved by God out of absolute space, Gassendi preserves a necessary dichotomy between space and matter; if God had created 'space,' then space would in some way be a substance, since it would be like matter derived from God. This conception of space allows matter free play within its boundaries and removes all questions of a medium that transmits mechanical operations (a problem in the physics of both Aristotle and Descartes), while also preserving God's absolute omnipotence.

Gassendi encounters a prickly theological point here, and he attempts to anticipate objections that the notion that anything in the universe could be uncreated undermines God's omnipotence. He maintains that since space "merely has the negative quality of allowing other things to occupy or pass through it," space is essentially 'nothing' and by definition 'nothing' cannot be ‘created’ by God or anyone else. Gassendi defines space as effectively ‘imaginary,’ since it is perceivable only “by analogy to the dimensions that appear to our senses.”[26] In other words, spatial dimensions can be perceived only when matter occupies or passes through space it is the matter that is perceived, and the dimensions themselves are extrapolated from matter by the mind. Unlike Descartes, however, Gassendi does not believe that dimensions exist only when occupied by matter; the fact that empty space cannot be perceived without matter does not mean it does not exist independently. Space, therefore, is fixed and absolute; this notion is perhaps the most easily recognizable legacy passed from Gassendi to Newton, and more than any other purely physical concept it distinguishes both philosophers from Descartes.

There is a very practical reason for Gassendi’s concern with the existence and nature of the vacuum. Without void, according to Gassendi, there can be no motion. This is because Gassendi imagines matter to be composed of Epicurus’ rigid, incompressible atoms which, if packed into a plenum, would be unable to move in any direction. In Aristotle’s plenist system, motion was accomplished by the rapid replacement of vacated space behind a moving body: an arrow, for example, was kept in motion by the bits of matter just behind it instantaneously filling in space left behind, because of nature’s *horror vacui*. To Gassendi, the notion of movement in a plenum was incoherent: how, he wondered, could one explain bodies changing place, or account for the compression of an elastic solid or fluid if every bit of space in the universe was occupied?

As for atoms themselves, Gassendi sticks very close to the account given in Lucretius’ poem *De rerum natura*. Essentially summarizing Lucretius, Gassendi posits several arguments relating to the nature of atoms: (1) that there must be a permanent stock of atoms in the universe they are neither created nor destroyed; (2) infinite division is impossible, because otherwise matter would dissolve away into tiny bits that could not form solid bodies; (3) there must be a distinction between matter and space (for reasons already discussed) and void is enclosed in matter; and (4) the existence of distinct species of objects is accounted for by the indestructibility and immutability of basic matter.[27] Atoms have three basic properties: size, shape, and motion, and every perceived quality can be accounted for by a combination of these properties. Gassendi also follows Epicurus in asserting that while atoms exist in a great variety of shapes and sizes, that variety is not infinite, otherwise there might exist some individual atom that is large enough to be seen. Gassendi has an additional reason to deny an infinity of shapes: if the shapes were infinite, so must be the number of atoms, and unlike Epicurus, Gassendi believes there is a finite quantity of atoms in the universe. This is for a strictly theological reason, since an infinite number of atoms would suggest that God (being the source and in a sense the ‘material’ of all matter) was spread out infinitely across the universe.[28] Gassendi does, however, deny Epicurus’ belief that atoms are endowed with an inherent faculty of self motion (which Epicurus claimed was 'downwards'); according to Gassendi "atoms are mobile and active from the power of moving and acting which God instilled in them at their very creation, and which functions with his assent."[29] But the phrase "functions with his assent" also suggests that Gassendi believes that God regulates the dispersion of atoms, and that unlike Descartes' God, he does not sit back to watch the proceedings passively.[30]

Gassendi's Nominalism

Gassendi developed his nominalist position primarily in two major treatises: the 1624 *Exercitationes* and the logical section of the *Syntagma*. Gassendi's first target is the notion that artificial methods of definition that produce categories such as 'genus' and 'species' somehow illuminate the inner natures of things. In Christianized Aristotelian (or Platonic) philosophy, true propositions may be made because God has composed nature as a hierarchy of essential categories, and logical operations draw on a preordained correspondence between the logical structure of the mind and the structure of the world. The 'book of nature' is directly translatable, from experience into concepts and from concepts into words. This correspondence exists, quite simply, because God made it so. The scientific investigator assumes and relies on this correspondence *a priori*; he does not create it through empirical investigation or necessarily verify it through experience.

Gassendi's epistemological approach thus joins his nominalism, moderate skepticism, and philosophy of logic, and must be viewed as a coherent and systematic program of natural philosophy. In the logical section of the *Syntagma* Gassendi delves deeply into the question of what truth is and how it can be known. As always, Gassendi begins by considering positions held in classical philosophy, eventually coming to rest on the opinions of the skeptics, who believed that nothing other than simple appearances could be known. Gassendi grants that he does not seek to understand the inner natures of things: "the truth in question here is not any general truth about existence."[31] Nonetheless, Gassendi continues, it is useless to debate whether anything exists at all; the testimony of the senses is enough to demonstrate existence, although it does not necessarily explain it.[32] Gassendi's proposal which relates directly to the problem of atoms is that if "the truth in question is hidden, lying concealed beneath experiences," we may "know it through some sign and whether we have a criterion by which we may recognize the sign and judge what the thing truly is."[33] The solution to this problem rests on Gassendi's understanding of two crucial concepts: "truth" and "sign."

In an earlier section of the *Syntagma*, Gassendi discusses the various kinds of truths sought in philosophy and discusses the question of hidden (or 'occult') truths. This passage is worth quoting at some length:

> Now, to speak specifically to the point of the truth sought by philosophy and the criteria by which it is possible to discern it, it must be remarked in the first place that some of the things whose truth can be sought after are manifest and some are hidden. And those which come to be known by themselves are *manifest* such as that it is daylight or daytime, such as the external appearance of things which strikes the eyes automatically and shows itself in our sight with no veil drawn over it; but the *hidden* things warrant a distinction into three types, as Empiricus shows so persuasively; indeed some are, or are said to be, totally hidden, some are naturally hidden, and some temporarily. The *totally hidden* are those which are such that in no way can they ever come within our grasp, for example, whether the number of stars is even. Things *naturally hidden* are those which cannot become evident by their own nature, or by themselves, but which we can nevertheless know and understand through something else, such as pores . . . Things *hidden temporarily* are those which, though they are evident by nature, are still hidden from us for the time being because of some obstacle, like a fire because a building is in the way . . . On the basis of this, the truth that philosophy seeks is not of manifest things since that is public knowledge; nor is it of totally hidden things since our ignorance of them is invincible, but of things hidden either naturally or temporarily.[34]

The task of philosophy, then, is to seek out truths that are *not* manifest, but this task is hampered by the limitations of the senses and reason. The existence of microscopic entities, like atoms or pores in the skin, may be inferred to a high degree of probability from their effects. The proof of their existence, however, is not *a priori*; it is derived in good empirical fashion through the senses. Gassendi's English epitomizer, Walter Charleton is more explicit about the explanation of supposedly 'hidden' phenomena. He criticizes the division between manifest and 'occult' phenomena in his translation and expansion of Gassendi's *Animadversiones* thus:

> the Schools; who too boldly praesuming, that all those Qualities of Concretions, which belong to the jurisdiction of the senses, are dependent upon Known Causes, and deprehended by Known Faculties, have therefor termed them *Manifest*: and as incircumspectly concluding, that all those Properties of Bodies, which fall not under the Cognizance of either of the Senses, are derived from obscure and undiscoverable Causes, and perceived by Unknown Faculties; have accordingly determined them to be *Immanifest* or *Occult*.[35]

Following Gassendi, Charleton is asserting that the division between manifest and occult is specious. He goes on to state:

> the *Sensibility* of a thing doth noe way praesuppose its *Intelligibility*, but that many things which are most obvious and open to the Sense, as to their *Effects*, may yet be remote and in the dark to the Understanding, as to their *Causes*: so on the contrary, doth not the *Insensibility* of a thing necessitate, nay, nor aggravate the *Unintelligibility* thereof.[36]

For Gassendi and Charleton, it is acceptable to reason by analogy from sensible effects or causes to insensible ones, as long as the 'signs' of those phenomena are read properly.

But what, according to Gassendi, constitutes an empirical 'sign'? In Chapter 5 of the logical section of the *Syntagma*, Gassendi considers this question in detail, surveying classical attitudes and beginning with a fairly broad definition: "In general terms anything that designates or signifies something different from itself can be called a sign, or if you prefer, any thing which, once it is known, leads us to the knowledge of something else."[37] Gassendi again raises the distinction between things that are 'naturally hidden' and those that are 'temporarily hidden,' and his concern is with the former category, which he believes to be the true subject matter of natural philosophy. His use of the term 'sign' in this case is not limited to what a modern observer might call a linguistic object, but rather he means an empirical device for uncovering knowledge that is hidden from direct view. Signs, for Gassendi, are heuristic mental tools for processing information about the external world. He says:

> An illustration of this is sweat as it indicates the existence of pores in the skin, for pores cannot be seen . . . Such also is vital action as it indicates the existence of the soul, and motion as it indicates the existence of the void . . . and other things of this sort.[38]

The 'signs' in these examples are essentially 'effects' that can be reasoned inductively to uncover hidden causes. But this process relies fundamentally on the senses, since "all knowledge which we have in the mind had its beginning in the senses," and "a certain sensible sign must come before the mind by which it is led to the knowledge of the thing lying hidden unperceived by the senses."[39] Gassendi is *not* describing abstract symbols in the mind that somehow allude to hidden properties of nature in the manner of Renaissance magi like John Dee or Oswald Croll; rather, Gassendi's signs are provided to the mind by experience, and all reasoning is based (at least by analogy) on sensible phenomena.[40]

Gassendi proceeds from this discussion to examples of a number of 'hidden' phenomena that are uncovered by empirical investigation of 'sensible' signs. He argues that the corporeality (and corpuscularity) of light are demonstrated by the reflection and refraction of light on glass: reflection and refraction are 'signs' that a solid body is colliding (in the former case) with particles of glass or passing through (in the latter) 'pores' in the glass. We cannot actually see corpuscles of light to verify this, but reason mediates sense experience to provide at least a probable knowledge of its nature.[41] Gassendi also provides examples from astronomy, arguing the phases of the moon (and of Venus) are signs of their spherical shapes, and the innumerable individual stars in the Milky Way are signified by "the perceptible sign of its filmy whiteness." Gassendi is particularly excited about the latter example, since it is a case in which a sign gave predictive knowledge of a particular phenomenon that was eventually verified empirically (by the telescope) during his lifetime. He wonders how many others "of those {phenomena} which are concealed in our time, which we perceive only through the intelligence, will one day also be clearly perceived by the senses through the agency of some helpful appliance thought up by our descendants?"[42]

Gassendi's epistemology as it relates to physics is therefore essentially semiotic: one 'reads' nature via empirical experiences as a mental language of signs. In the next section, I will discuss logic and the mental manipulation and representation of 'signs' by language and mathematics, but it is worth noting here that Gassendi privileges the senses before reason when it comes to obtaining natural knowledge. Gassendi was not such a naïve empiricist that he denied reason any role in scientific investigation, which being "superior to the senses, can correct the perception of the senses."[43] But according to Gassendi's epistemology, the senses supply the raw material for the intellect to digest, and being unmediated by reason, sense data is therefore less likely to be the source of error. The senses are essentially passive receptors: Gassendi explains vision, smell, hearing, taste, as the collision of particles with organs of sense.[44] While he grants that those organs can be faulty or in error (damaged by injury, for example), they are the closest humans can come to unmediated experience of nature; they are the first step in translating experience to knowledge. The intellect then is at least two steps removed from nature (more, if it is acting on data not directly provided by the senses, such as a second hand report), and when the intellect reasons on experience it is essentially translating material that has already been translated by the senses. Each additional translation is an opportunity for error to be introduced, and each judgement the intellect makes draws the mental picture further away from unmediated sensory apprehension. As we will see in the next section, the connection between knowledge and certainty for Gassendi is a tenuous one, and the very inability of human beings to represent knowledge without translating from experience precludes the possibility of establishing natural philosophy founded on absolute truth.

Constructivism

Gassendi's views concerning logic can be traced back to his earliest writings, notably the *Exercitationes* of 1624, before his fascination with Epicurean atomism began. As Gassendi noted in the introduction to this work in the *Opera*, his critique of Aristotelian modes of logic was initially influenced by the radical skeptics of the Pyrrhonist tradition, who questioned the basis for any knowledge of nature at all. But he also claims that even at this early stage he tried to steer a middle course, and he informs the reader that he wishes to place himself neither on the side of the Aristotelians nor on that of the "dogmatics," among whose number he counts Sextus Empiricus and Pyrrho of Ellis.[45] Indeed, the picture Gassendi paints of his skepticism is at the same time both more moderate, and in a sense also more radical than dogmatic Pyrrhonism, because, unlike the dogmatists, Gassendi acknowledges that even his skeptical epistemology must be mitigated by skepticism itself. He writes:

> Since I cannot really persuade myself that the truth of things can be perceived by mortal men, I am far from wishing to sell my wares; and the things that I appear to be asserting here are not meant to be taken as established facts.[46]

Gassendi's position is that the essential truth of nature cannot be perceived by humans, but he also acknowledges that because beliefs are uncertain, he may be wrong about even that.

In the *Exercitatones*, Gassendi is primarily concerned with defining the goals and limits of logic and with criticizing Aristotelian modes of logic for (1) failing to meet those goals; and (2) attempting to transcend their limits. Gassendi first attacks the notion that artificial methods of definition that produce categories like 'genus' and 'species' somehow illuminate the inner natures of things, and that nature is divided into sets of actually existing 'universal categories,' which can be apprehended directly by the mind.[47] Here Gassendi follows the nominalists, and rejects the notion of universal categories entirely. Definitions, according to Gassendi, produce only artificial classes, since they rely on judgements to sort experience into categories. In the *Exercitationes*, he argues "all universality lies in the domain of concepts or words," because "the understanding forms a statement and a predication concerning things as it conceives and names them."[48] This is a position echoed in his declaration that there is "no universality outside of thoughts and names."[49] Gassendi's nominalism is also related to his belief that individual experiences of appearances do not produce universal or consistent mental representations of natural objects. This is exemplified in the variance between accounts of a particular phenomenon by different observers:

> since there are so many different appearances of one and the same thing, and since so many different judgements are passed upon it, both by different animals and by different men, as well as by one single man, what other conclusion remains except that we cannot know what anything is like according to itself or its own nature, but only how it appears to some men or to others?[50]

How, Gassendi wonders, can two people be certain that they are describing the same thing when we use particular words to define an object of knowledge? We derive experiences from the senses, but we have no way of knowing what it is like to have another's sight or taste, nor (given disagreements about particular qualities of things) do we have any way of verifying that our own senses are not in error. More importantly, since logic is used to communicate knowledge and, as Gassendi notes approvingly, according to Cicero logic "make[s] clear what is obscure by translating it into other terms" Gassendi despairs of finding a common ground on which to base representations of experience.[51] Individual human minds form 'mental pictures' of their experiences, but without putting those pictures into words and exposing them to a further level of translation and distortion there is no way to communicate them to others. Having thus attacked the foundations of positive knowledge, Gassendi sounds his most pessimistic note: "And so I will conclude here . . . [with] a certain confirmation of the statement *quod nihil scitur* [that nothing is known]."[52]

If nothing can be known, one might wonder, what is the point of pursuing natural philosophy? Having attacked the foundations of knowledge Gassendi proceeds, in the *Exercitationes* and much later in the *Institutio Logica*, published in the *Opera*, to try to re construct an epistemology that takes a *media via* between radical doubt and certainty. For Gassendi, truth and certainty are logical terms rather than absolutes, but this does not preclude the possibility of advancing knowledge. Natural philosophy, however, is framed as a logical reconstruction of nature rather than a passive 'reading' of her transparent signs, and linguistic construction is its chief mode of representation. Gassendi begins *Institutio Logica* with the statement "Logic is the art of correct thinking," which recalls Ramus, whom Gassendi cites a number of times in the *Opera* as a major influence on his thought. Gassendi's definition reveals his linguistic conception of logic, an art which "takes its name from the Greek word logos which means 'speech,' since thinking is nothing else but an inner conversation which the mind holds with itself."[53] Logic, Gassendi is saying, is a construct that follows rules similar to those of grammar and, when followed correctly, produces 'correct' thinking, just as grammar produces 'correct' speaking.

This comparison also sheds light on Gassendi's theory of signs as they relate to mental operations. As we have already seen, Gassendi believes that physical objects are represented as 'signs' by the intellect, which are constructed from experiences of nature. This involves a multi step process of translation: first the senses translate a natural object to a sense impression (the sensation of a particular image, or taste, or smell, etc.); next the mind translates the raw sensation into an 'idea' of the sensation (the sensation is identified as having particular qualities); finally the 'idea' is correlated with a particular word ('red,' 'bitter, 'acrid') that can be used by the mind in logical operations or for communication with another person (yet another translation). An 'idea,' according to Gassendi, is "that image which is present to the mind, indeed is thrust before it almost," which is also called "form," "concept, preconception, anticipation, innate concept . . . conception and phantasm" by others.[54] But it should be very clear that, for Gassendi, an 'idea' is nothing like one of Plato's forms; it does not exist apart from experience, since "every idea which is held in the mind takes its origin from the senses" and "every idea either comes through the senses, or is formed from those which come through the senses."[55] Constructivism enters when particular ideas are grouped together to form compound ideas in the mind, such as

> when from the ideas of mountain and gold [i.e. the *impressions* of those physical objects] the mind pictures the idea of a mountain of gold; when from the ideas of man and horse it gets the idea of centaur . . . and so on.

This process Gassendi terms "unification." Ideas can also be connected to form general categories, as when the mind takes individual ideas and

> by examining the ideas individually and separating out that which all have in common, at the same time disregarding or ignoring mutual differences, takes as a common, universal and general idea that which has been thus abstracted and which contains nothing that is not common . . . and this is what is termed the genus.[56]

Like Ockham, then, Gassendi views categories (or universals) as artificial mental constructs. This is consistent with his nominalism, and Gassendi asserts that these universal or general ideas are not ontologically real entities: "all the things which are in the world and which are able to strike the senses are singulars."[57] Although the mind constructs compound and general ideas of objects, this is an artificial process. This does not, however, lead Gassendi to reject the possibility of obtaining knowledge. Gassendi quotes the Stoic dictum *quod nihil scitur* only to reinforce his claim that "men do not know the inner nature of things, or their so called real essences"; it is basically a rhetorical device, designed to point out the limits of human knowledge, not its impossibility.[58] Gassendi's solution to this problem is essentially to propose an extreme form of epistemological constructivism: abstracted general categories have value in philosophy because they are constructed by the human mind, and therefore may function in logical propositions whose rules are defined artificially.

Gassendi's attitude towards the limits of knowledge is reflected in his inductive, *a posteriori* approach to investigating nature. Gassendi's ontology also mirrors his epistemology: because he is (essentially) a materialist, he believes only in knowledge of appearances; because he can only know appearances, he is a 'physical' nominalist (every object is an individual); this leads him to epistemological nominalism (only individuals can be known); because of his nominalism he bases concepts like truth and certainty in linguistic constructivism. The 'book of nature' metaphor is not inappropriate here, but Gassendi's semiology is constructivist: natural objects are *perceived* as signs because that is the only way humans can recognize them; they are ordered and categorized according to an artificial grammar of signs, and are ultimately translated into words. Nature is not decoded, it is encoded by the sensory, intellectual, and representative faculties, and knowledge is artificial but also constructive.

Mathematics

Gassendi extends his nominalism and constructivism to mathematics as well. Later in the *Exercitationes*, he concludes "that whatever certainty and evidence there is in mathematics is related to appearances, and in no way related to genuine causes or the inner natures of things."[59] This is the same caution he makes when he discusses the limits of knowledge in language and logic: propositions may only be said to be certain or true insofar as they reflect our imperfect perceptions of empirical data, and not in relation to some absolute standard of truth or existence.[60] Indeed, as Gassendi continues, mathematics must be explicitly concerned with matters of experience, since "the moment you pass beyond things that are apparent, or fall under the province of the senses and experience, in order to inquire about deeper matters, both mathematics and all other branches of knowledge become completely shrouded in darkness."[61] Clearly Gassendi does not extend special epistemological privileges to mathematics, nor does he grant mathematics a unique role in scientific investigation; mathematics, like all other branches of knowledge, must be concerned only with knowledge of appearances and individuals. This means that Gassendi's view of mathematics is, like his view of language, nominalistic. Mathematics, being a human interpretive faculty, seeks to make general statements about objects of knowledge. Because, however, the mind only has access to individuals, that generalizing process is artificial. Just as words are general terms — based on the experience of individuals — that have no real existence in nature, so must mathematical objects be "considered in actual things," since "as soon as numbers and figures are considered abstractly (a way they have never been in existence), then they are nothing at all."[62]

Gassendi concludes his discussion of mathematics in the *Exercitationes* by noting that a geometrical object, such as a triangle, is grouped by the mind into a general class even though it is based on knowledge only of particulars. As he states:

> if mathematics makes some proof, for instance about the triangle, it does not name this triangle or that one; but yet it understands this triangle and that one, not singly, but in conjunction with all others. In fact, however, if it did not base its conclusions upon triangles appearing in some material form, it would only be chasing chimeras since no other triangles but these can exist.[63]

For geometry to have any usefulness, it must, according to Gassendi, deal with general classes of objects, but paradoxically geometrical objects only derive their validity as they are observed in actual objects by the senses. It is the same with Gassendi's conception of language: words exist only as they are abstracted from sensory data, but that very abstraction prevents them from ever capturing things as they 'actually are.' Gassendi (and, in similar terms, Hobbes) argued that because mathematics relies on abstraction it is essentially a 'grammar' for representing imperfect perceptions of physical objects. This position reflects Gassendi's commitment to a form of mathematical constructivism, which attributes a great deal of 'certainty' to mathematics, but whose certainty is based on mathematical rules and objects that are constructed by human reason. Gassendi's nominalism and mathematical constructivism thus depart from Cartesian and Galilean philosophy of mathematics by denying that mathematical objects (and similar abstractions) are ontologically real.[64]

Gassendi's interest in the epistemological and ontological foundations of mathematics continued in the years following the *Exercitationes*. Lynn Joy describes Gassendi's involvement in a debate over the existence of indivisible magnitudes that circulated among a group of philosophers close to Marin Mersenne in the mid 1630s. Whereas most of the participants chose to analyze indivisibles as if they were mathematical points, Gassendi argued from the position of physical atomism, and his concerns about mathematical abstraction are in evidence. Joy quotes from a letter to Mersenne in which Gassendi raises several doubts about the validity of applying mathematical demonstrations to solve what he considers a purely physical problem, and she states that Gassendi's skepticism about the reality of abstractions separated him from the other participants in the debate.[65] Joy also connects this attitude to Gassendi's nominalism, and cites Gassendi's belief that "anyone attempting to give a mathematical description of a physical state of affairs should always bear in mind the fact that such a description is applied to individual physical objects with varying degrees of accuracy."[66]

Gassendi's response to Mersenne in this instance offers not just the standard empiricist objection that the senses are superior to the intellect, but also a more penetrating skepticism about the existence of mathematical objects at all. In explaining his position, Gassendi drew an analogy between realist interpretations of mathematical astronomy and the status of physical indivisibles:

> I mention this [the debate over Ptolemaic versus Copernican systems] only in order that you understand that I say nothing absurd when I hold as mere hypotheses the points, lines, and surfaces defined by the mathematicians every one of which could be made about nonexistent things.[67]

As Gassendi, a student of the history of philosophy, well knew, the Copernican and Ptolemaic systems can be made to be mathematically (and predictively) equivalent. In the end, it is only the evidence of the senses in the case of cosmology observations of phenomena like the phases of the moon and Venus that decide the case. The 'truth' of mathematics is not the final arbiter of physical questions. Relating this back to the case of indivisibles, Gassendi is suggesting that a geometrical construct (such as a point) that has no basis in experience cannot be used to signify an actual physical state of affairs.

Gassendi and Descartes

Gassendi's mathematical philosophy is also evident in his published objections to Descartes' *Meditations on First Philosophy* in 1642. Descartes invited his close friend Mersenne to solicit responses to the ideas in his book, as Descartes later put it, "not only by the Doctors of the Sorbonne, but also by all other learned men who would take the trouble to scrutinize them."[68] Along with Gassendi, six other notable scholars complied, including Mersenne himself, Hobbes, and the logician and grammarian Antoine Arnauld. All of these 'objections' were published in subsequent editions of the *Meditations*, along with Descartes' replies. We will have occasion to examine Hobbes's set of objections in the next chapter, but it should be noted that the fifth set Gassendi's was the longest, most critical, and provoked the most defensive response from Descartes. Descartes as he himself reports in the "Appendix to the Fifth Set of Objections and Replies" asked that Gassendi's objections be removed from the French translation on the grounds that they were "extremely long," and because they contained not "a single objection which those who have some slight understanding of my *Meditations* will not, in my view, be able to answer quite easily without any help from me."[69] Indeed, Gassendi eventually replied to Descartes' responses with a new set of objections, all of which were collected and published separately as *Disquisitio Metaphysica adversus Cartesium*. Given their length, I will not attempt a point by point analysis of this exchange; instead, I will characterize the essential points of disagreement between Descartes and Gassendi over questions concerning representation and mathematics, and give a flavor of Gassendi's responses.

Descartes, like Gassendi, drew close parallels between the mental operations performed in logical and linguistic arguments and those in mathematics. The difference, of course, is that Descartes believed in the reality of mental abstractions, and in the innate correlation between those representations and the physical world. In his examples, Descartes turns explicitly to mathematics and geometry for evidence of the certainty of human cognition; indeed, mathematics is the paradigm for such examples, as it displays the power of the human mind. Descartes claims, for example, that regardless of the physical existence of any perfect geometrical figure (such as a triangle), because he can imagine that figure having certain determinate properties, there must be "a determinate nature, or essence, or form of the triangle which is immutable and eternal, and not invented or dependent on my mind."[70] This suggests that not only are humans able to clearly and distinctly perceive the essences of things (even incorporeal things), they are also able to conceive and represent those essences without distorting them.

Descartes' mathematical ontology naturally complements his physics: Nature is, for Descartes, entirely abstractable, and the only 'real' properties in physics are extension and motion, which are treated as idealized rather than purely physical concepts. In the *Principles*, Descartes cites several arguments demonstrating the "impossibility of atoms," stating:

> We also know that it is impossible that there should exist atoms, that is, pieces of matter that are by their very nature indivisible. For if there were any atoms, then no matter how small we imagined them to be, they would necessarily have to be extended; and hence we could in our thought divide each of them into two or more smaller parts, and hence recognize their divisibility. For anything we can divide in our thought must, for that very reason, be known to be divisible; so if we were to judge it to be indivisible, our judgement would conflict with our knowledge.[71]

This argument against atoms begins with a premise that extension implies distinct parts that follows Aristotle's critique of Democritus, but goes even further: not bothering to draw any distinction between actual division and theoretical division, Descartes cites the imagination of division as the *proof* that it is physically possible. The legitimacy of abstracting from nature is necessary both for Descartes' ontology and his philosophy of mathematics. Mathematical concepts are in a sense not just 'representations' of physical quantity (as they are for Gassendi), but rather real essences that have existence apart from physical extension. This is why, in algebra, Descartes was able to move away from an explicitly numerical or arithmetic conception in favor of a more general system in which 'number' was freed from its connection to actual physical quantity. In algebraic equations, general notation allows operations that are valid for any and all imagined quantities, including irrationals and imaginary roots, because the vocabulary of mathematical objects is expanded to include anything that the mathematician can conceive.[72]

But in Descartes' philosophy algebra is more than just a 'grammar' for producing logically consistent statements: it captures real truths about the actual state of affairs in the world. Jacob Klein has defined Descartes' method of *mathesis universalis* as "a symbol generating abstraction," which fits the more linguistically oriented approach many seventeenth century mathematicians (nominalist or not) were beginning to favor (and which, in Klein's analysis, had roots in François Vieta's symbolic algebra).[73] However, Klein also explores how Descartes' *mathesis* could produce a 'true physics,' and determines that, for Descartes:

> The 'service' rendered by the imagination depends, therefore, on a 'real,' and not a 'figurative,' 'rendering' of the corporeal world; the imagination always represents precisely that within the corporeal world which *really* constitutes its true nature, its 'substance,' its 'corporeality' namely 'figurate' extension as such. . . . [This is] why the imagination can guarantee the ability of the *mathesis universalis* to grasp the structure of the 'true world.'[74]

Descartes explores the relationship between mathematics and physics a letter to Mersenne of 1630, and concludes that mathematical 'truths' are established by God and are written into the fabric of nature, much as physical laws are. In a striking passage, he compares divine mathematical laws with the laws of a temporal king:

> But in my physics I will not touch on certain metaphysical questions, particularly this one: That the truths of mathematics, which you [Mersenne] call eternal, are established by God and are entirely dependent on Him as are all the rest of the creatures [*tout le reste des creatures*]. It is in fact the same to speak of God as a Jupiter or Saturn who are subject to [the River] Styx and to destiny, as to say that these truths are independent of Him. Do not fear, I urge you, to argue and publish everywhere, that it is God who establishes these [mathematical] laws in nature, just as a king establishes the laws in his kingdom. Now there is no one [law] in particular that we cannot comprehend if our intellect makes itself consider it, and they are born in our minds [*mentibus nostris ingenitae*] as would a king imprint his laws in the hearts of all his subjects, if he had the power to do it.[75]

It is particularly important to note here Descartes' claim that the ability to 'read' mathematical law is an "inborn" faculty, which supports his claim elsewhere that the mind is directly able to apprehend mathematical truth and also harkens back to Plato's doctrine of reminiscence. The difference between Descartes' conception and Gassendi's cannot be overstated. For Gassendi, physics is a necessarily uncertain empirical exploration of the natural world. For Descartes, "the only principles which I accept, or require, in physics are those of geometry and pure mathematics; these principles explain all natural phenomena, and enable us to produce quite certain demonstrations regarding them."[76]

In a letter to Clerselier attached to the "Appendix" to the fifth set of objections, Descartes mockingly summarized Gassendi's response:

> many people of great intelligence [i.e. Gassendi] think they clearly see that mathematical extension, which I lay down as a fundamental principle of my physics, is nothing other than my thought, and hence that it does not and cannot have any subsistence outside of my mind,

> being merely an abstraction which I form from physical bodies. And they conclude that the whole of my physics 'must be imaginary and fictitious, as indeed the whole of pure mathematics is, whereas real physics dealing with things created by God requires the kind of matter that is real, solid and not imaginary.'[77]

Nonetheless, he reports satisfaction that his critics "link my physics with pure mathematics, which I desire above all that it should resemble."[78]

Not surprisingly, Gassendi objected to a number of Descartes' key metaphysical positions, beginning with a challenge to Descartes' belief in 'real essences' and his strong claims for rational certainty. In the *Meditations*, Descartes argued that the sole cause of errors in the intellect are 'preconceived notions' beliefs that cause the mind to draw unsound judgements about the objects it considers. Gassendi employing the systematic logical method of critique that he used throughout his objections challenged Descartes' claim that the mind is like a "*tabula rasa*," able to form judgements without the influence of any prior notions. Gassendi focused on two claims of Descartes' that he found to be contradictory: that the mind is able to perceive essential principles of nature without the aid of the senses, and that the mind contains no preconceived notions. "Clearly," Gassendi argued,

> if it [the mind] has no preconceived notions, then it does not have any principles; for principles, as they are here understood, are statements; and statements are kinds of judgements in which something is either affirmed or denied. Hence these principles will be judgements, and inasmuch as they are conceived in advance, they are preconceived notions. Therefore, if the mind has no preconceived notions in it, neither will it have any principles from which it may deduce something.[79]

It is obvious that Gassendi understands 'principles' to mean something other than what Descartes intends: whereas for Descartes principles are timeless, idealized truths, for Gassendi, they are logical constructions that are judged true or false depending solely upon whether they meet the criteria set forth in their definitions.

When he reaches Descartes' discussion of geometry and mathematics, Gassendi's nominalism surfaces clearly. In the fifth Meditation, Descartes had argued that geometrical demonstrations of objects such as triangles are proven by knowledge of the existence of God. Because God exists, and has granted us the power of clear perception, our perceptions of geometrical objects pure objects, that is, unmediated by the senses must be of eternal truths; furthermore, our intellectual awareness of geometrical objects gives those objects existence regardless of their ever having physically existed in nature. Gassendi responds, however, by claiming that while a figure having three angles and three sides "will not fail to be a triangle" simply because that is the word's definition, the word 'triangle' signifies nothing without being tied to physical experience: "things not yet created and having no existence, but being merely possible, have no reality and no truth."[80] Gassendi also doubts Descartes' ability to have 'understanding' of more complicated geometrical figures (such as the chiliagon, a thousand sided figure) in the way Descartes claims, and he accuses him of confusing 'imagining' with 'understanding.' Gassendi grants that one may call up a mental picture of a triangle, and 'understand' that this figure is connected with a particular word and definition, but he denies that Descartes' mind can "grasp distinctly" a thousand sided figure, or that he can truly understand whatever tortured representation of that figure his mind is able to create. At most, Gassendi argues, one can "perceive that the word 'chiliagon' signifies a figure with a thousand angles"; however, "that is just the meaning of the term, and it does not follow that you *understand* the thousand angles of the figure any better than you *imagine* them."[81] Ultimately, Gassendi concludes, all knowledge including mathematical knowledge must be tied to the experience of particulars in order to be considered to have real existence. In a denial of one of Descartes' stated principles of physics, Gassendi announces that "the subject matter of pure mathematics including the point, the line, the surface, and the indivisible figures which are composed of these elements and yet remain indivisible cannot exist in reality."[82]

From his response, it is clear that Descartes recognized the philosophical basis for Gassendi's attack, though he disingenuously pretended that Gassendi's "purpose has rather been to bring to my attention the devices which might be used to get round my arguments by those whose minds are so immersed in the senses that they shrink from all metaphysical thoughts."[83] Later, in response to Gassendi's objections to his philosophy of geometry, Descartes argues that

> unless you are maintaining that the whole of geometry is also false, you cannot deny that many truths can be demonstrated of these essences; and since they are always the same, it is right to call them immutable and eternal. The fact that they may not accord with your suppositions about the nature of things, or with the atomic conception of Democritus and Epicurus, is merely an extraneous feature which changes nothing; in spite of this they undoubtedly conform to the true nature of things established by God.[84]

Joy has argued that the failure of Descartes and Gassendi to meaningfully communicate was due to methodological differences Descartes was unable to parse Gassendi's logical critiques and humanist methods. But my interpretation of this exchange is that the problem was more fundamental. Here I am inclined to follow Lennon, who maintains that Gassendi's conception of "the identity of idea and object demands an ontology incompatible with Descartes's mathematical realism, his Platonic account of the material world."[85] Indeed, Gassendi himself characterized Descartes' arguments about triangles as Platonic and questioned whether “just because these and other splendid and Platonic things have been said, have they been splendidly and truly proven?”[86] Gassendi’s argument with Descartes, in other words, portrays two major interpretive modes in the philosophy of science in the seventeenth century one embracing linguistic and mathematical realism and the other constructivist nominalism and the protagonists in the debate explicitly recognized the grounds on which the dispute lay (even if, in the case of Descartes, the two did not always respect one another’s positions). We, as modern observers, may characterize Descartes’ philosophy as something other than Platonism, but to Gassendi it clearly contained elements of idealist essentialism that were incompatible with his own empirical and materialist views. The failure of Gassendi and Descartes to conduct a meaningful exchange of views does indeed betray an incommensurability of world views, but they are not between ‘scientific philosophy’ and Renaissance humanism as Joy claims, but rather between the belief structures of nominalism and metaphysical realism.

As is indicated by the preceding discussion, Gassendi was, during his lifetime, regarded as a figure of sufficient importance to merit the inclusion of his objections in one of Descartes’ premier works. This does not necessarily indicate, of course, that Gassendi’s fame equaled Descartes’; it may be argued, however, that although Gassendi’s reputation was not as widespread as Descartes’, his views were extremely persuasive among an important group of natural philosophers.[87] In the next chapter, I will discuss evidence for the influence of Gassendi’s epistemology on Hobbes and Locke, which has remained a fairly contentious issue among modern historians. There was certainly nothing approaching a Gassendist ‘school’ during the seventeenth century, but I am arguing that there was a ‘tradition’ of nominalism that produced recognizable positions about the knowability of natural phenomena and the representation of concepts in language and mathematics that were crystallized by Gassendi’s work. Many of the physical questions raised by atomism as well (such as the limits of divisibility) translate to epistemological questions in mathematics (such as the use of infinitesimals and infinite series) that figured into the work of later figures such as Newton and Berkeley. Although solutions to these questions varied widely, their persistence demonstrates that the issues touched on by Gassendi were not of merely passing interest. The problem of reconciling perception with cognition and representation remained perhaps the central epistemological question of the scientific revolution, and it was a question that was not settled decisively in that century, or even in our own.

3 British Empiricism, Nominalism, and Constructivism

If I were arguing that Gassendi inaugurated a particular 'school' of mathematical philosophy, it would be fair to ask whether Gassendi's nominalistic mathematics itself had any real or lasting impact on later beliefs and practices. Although his philosophy was certainly not without influence Gassendi was read and admired by Hobbes, Barrow, and Newton, whose positions will be examined presently I am not arguing for a direct chain of causal influence between Gassendi and these other natural philosophers. Rather, I argue that Gassendi's position on foundational questions in the philosophy of mathematics is representative of a distinctive approach to these questions during the seventeenth century an approach that is hallmarked by commitments (in varying proportions, to be sure) to nominalism, mathematical/epistemological constructivism, and theological voluntarism. This constellation of beliefs had a currency in seventeenth century intellectual culture that was broader than the views of any one particular philosopher, but it has not been linked explicitly to mathematics in scholarly literature.

By the middle of the seventeenth century, many of the epistemological issues popularized and debated by Gassendi, Descartes, Mersenne and others during the first half of the century had reached a new level of currency within the general European scientific community. Discussions about the method, organization and representation of science and scientific knowledge became more common as greater financial and institutional support became available to natural philosophers, and as scientists outside universities sought to organize themselves into coherent communities and to standardize their practices. Organizations such as the Royal Society in London and the French *Académie* at Paris, which formed after 1650, grew out of the more informal correspondence networks and salons that preceded them, and their proceedings and official publications greatly aided the production and transmission of new ideas in astronomy, chemistry, mechanics, mathematics and medicine.[1] The need for generally accepted principles of epistemology and ontology became more pressing as mechanical philosophers sought to justify and standardize their scientific approaches. The growth of experimental practices and techniques, on the one hand, and greater mathematical sophistication, on the other, opened new areas of discovery and exploration, but also presented greater opportunities for dissent, discord and confusion in the scientific community.

The epistemological issues discussed in the last chapter had, in other words, entered the mainstream of scientific discourse by the 1650s and 1660s. The year of Gassendi's death 1655 was also the year of publication of Thomas Hobbes's *De Corpore*, the work he intended as the summary of his general philosophical system. This treatise considered many of the issues raised by Gassendi and Descartes on the subjects of logic, representation, and language, and argued against the rationalistic approach of Descartes and his followers. Over the next few decades a series of works on logic and language were produced on both sides of the Channel, and most were concerned explicitly with applying epistemological principles to issues raised by the new approaches to mathematics and the physical sciences in natural philosophy. Among the most notable examples were the works on grammar and logic of the Port Royal logicians, whose *Logic*, published in 1662, attempted to give Descartes' philosophy a rigorous logical foundation. Also important were the works of so called universal language planners such as John Wilkins, John Webster and Seth Ward, who were concerned with clarifying the means by which scientific knowledge could be represented and communicated with philosophical certainty. The end of the century saw the publication of two of the most important works in the history of English natural philosophy: Isaac Newton's *Philosophiae Naturalis Principia Mathematica* of 1687, and John Locke's *An Essay Concerning Human Understanding*, published in 1689. The turn of the eighteenth century also saw the production of George Berkeley's *A Treatise Concerning the Principles of Human Knowledge*, in 1710. These last three works, as I will argue, had a significant basis in nominalism and constructivism.

As mathematical natural philosophy grew in sophistication and explanatory power over the course of the seventeenth century, however, new concerns about the ontological status of mathematics took on greater urgency. Nominalist constructivists, with their emphasis on the arbitrary or conventional nature of signs (linguistic and mathematical) objected most strenuously to the notion that signs had a necessary connection to objects in the world. Abstraction of particulars to form universal or general statements was understood as a human process of mental construction that had an inherently provisional and *ad hoc* basis. Language, as Gassendi suggested, is in a sense a kind of word game. Ironically, though, it was precisely the notion that signs are arbitrary or conventional signifiers that gave power to new techniques in analytic geometry and algebra that led to the calculus, introducing a tension in constructivist epistemology. This tension was a central feature of Hobbes's philosophy of language and mathematics.

Hobbes

More than one historian has remarked that Hobbes's theory of cognition and representation is explicitly nominalistic. According to the historian of linguistics G. W. Padley, Hobbes was "the exemplar of this tendency to extreme Nominalism" which has its roots in the skeptical philosophy of the late scholastic period, "which Hobbes did more than any other philosophical writer to hand on to modern times.[2] Padley's assessment is supported by Douglas Jesseph, who contends that Hobbes's philosophy "goes beyond the nominalism of his predecessors," including that of the 'original' nominalist, William of Ockham.[3] Indeed, Hobbes's theory of language and its role in concept formation are quite similar to Gassendi's, and his nominalism is explicit. Furthermore, as Amos Funkenstein observes, constructivism was an essential feature of Hobbes's epistemology: "Hobbes believed that 'truth' and 'the constructible' are synonymous."[4]

Hobbes's philosophy was influenced by his intellectual circle, and over the course of his life he corresponded and interacted with many of the leading natural philosophers of his day. Early in his career Hobbes came into the service of the powerful Cavendish family in England, and this led to an association with the Earl of Newcastle and members of the scientifically minded 'Newcastle Circle' in the 1630s and 1640s, who were among the first to promote mechanical philosophy in England.[5] As a royalist supporter who spent much of his adult life in Paris, Hobbes came to know Mersenne and was included in Mersenne's wide group of correspondents, which brought him into contact with Descartes and Gassendi. Although Hobbes never fully accepted Gassendi's system of Epicurean atomism as a physical ontology, he was clearly aligned with Gassendi on epistemological issues against the Cartesians; this is evidenced directly by Hobbes's participation in the *responsa* to Descartes' *Meditations* and in subsequent correspondence.

It is obvious from a number of letters of this period that beyond deeply respecting Gassendi as a philosopher, Hobbes also considered him a personal friend.[6] Samuel Sorbière's introduction to Gassendi's *Opera* relates an anecdote in which Gassendi, on his deathbed, "had received a copy of *De corpore* from Abraham du Prat, and greeted it with a kiss."[7] The more pressing and complicated question is whether Hobbes owed a true intellectual allegiance to Gassendi. As members of the same loose intellectual circle that gravitated around Mersenne and maintained ties through correspondence with Mersenne, Sorbière, du Prat and others, Gassendi and Hobbes shared a commitment to the 'new philosophy,' insofar as it meant establishing a system of natural explanation that challenged the traditional interpretations of the schools. In the eyes of their mutual friends, however, Gassendi and Hobbes were seen as advocates of the same general principles, and in any event, Gassendi and Hobbes were unquestionably familiar with each other's work.[8] Their contemporaries, who thought it logical to associate the ideas of the two men, recognized this fact. One uniting factor was their opposition to Descartes, which forms a convenient backdrop against which to compare their views. But I will argue that despite 'superficial' ontological disagreements, Gassendi and Hobbes shared a commitment to the core epistemological values of nominalism and constructivism with respect to the acquisition and representation of knowledge. Furthermore, Hobbes's philosophy of mathematics is in fact an attempt to provide a foundation for a position first expressed in Gassendi's work: where Gassendi claims that universal notions are artificial compounds constructed in the mind, Hobbes extends this 'constructive' approach explicitly to mathematical objects, providing mathematics a basis for certainty while preserving a nominalistic theory of representation.

Hobbes's Theory of Representation

Hobbes's two major works on scientific epistemology, from which his theory of representation is derived, are *Leviathan*, published in 1651, and *De Corpore*, which first appeared in 1655. It is on the basis of the former that Hobbes's reputation as a political philosopher was cemented, but the first of four parts of *Leviathan*, titled "Of Man," are predominantly an account and description of human mental faculties. *De Corpore*, which was subsequently (and with Hobbes's approval) published in English translation as *Elements of Philosophy*, is a far more general work in which Hobbes presents a complete philosophical system based on a materialist ontology and an empiricist epistemology.[9] It is worth noting, however, that Hobbes reported in a letter as early as 1647 that he had nearly completed the first part of his 'Philosophy,' which concerned physics.[10] We can conclude, therefore, that Hobbes's philosophy of science was fairly well developed prior to 1650, which is borne out by the fact *De corpore* generally agrees with the epistemology of *Leviathan*.

Many of the concepts introduced in *Leviathan* are repeated and amplified in *De corpore*, but *Leviathan* provides a good introduction to Hobbes's general scheme of representation. Essentially, he accepted that all ideas formed in the mind originate in the senses. Hobbes's scheme is noteworthy, though, for the lengths it goes to speculate about the physical mechanism of cognition. Sensation, according to Hobbes, begins with the physical impression of the qualities of objects in nature on the organs of sense: the 'pressure' that results, "by the mediation of Nerves, and other strings," is transmitted to the brain, where an 'imagination' of the object is formed.[11] Imagination or the 'image' formed of an object is "nothing but *decaying sense*," since the mind 'fancies' that it still apprehends an object that is no longer directly before it. In other words, the sensations produced by contact with a physical object are retained in the brain, but over time these impressions are diminished and the correspondence between the image and the original object fades. This Hobbes terms 'memory,' and he states "*Imagination* and *Memory* are but one thing, for which divers considerations hath divers names."[12]

The culmination of the process of sense perception is "understanding," which follows directly from memory. According to Hobbes, there are two kinds of understanding: "The Imagination that is raysed in man (or any other creature indued with the faculty of imagining) by words, or other voluntary signes, is that we generally call *Understanding*; and is common to Man and Beast," while "that Understanding which is peculiar to man, is the Understanding not onely his will; but his conceptions and thoughts, by the sequell and contexture of the names of things into Affirmations, Negations, and other forms of Speech."[13] This latter conception of understanding is where Hobbes treats language and other kinds of symbolic representation. In order for the mind to make judgments about experience (as opposed to passively receiving sensory input, which is what lower animals do), it must have some means of connecting and communicating objects of experience: "By *Consequence*, or TRAYNE of Thoughts, I understand that succession of one Thought to another, which is called (to distinguish it from Discourse in words) *Mentall Discourse*."[14] Here Hobbes describes mental discourse in explicitly empirical terms:

> In summe, the Discourse of the Mind, when it is governed by designe, is nothing but *Seeking*, or the faculty of Invention . . . a hunting out of the causes, of some effect; or of the effects, of some present or past cause.[15]

The method of science is to observe nature to make causal connections, and this process is exactly mirrored in the act of cognition.

For this the mind needs signs, or "marks," since "some sensible tokens are necessary, to which past thought can be reduced, and which can be registered in their own orders as it were." These "marks" are "sensible things employed by our own decision, so that at the sensation of these things, thoughts can be recalled to the mind, similar to those thoughts for the sake of which they were summoned."[16] But in order for these tokens to aid in the communication of information between individuals, there must be a guarantee that marks intelligible to one person will be understood in the same way by another. Hobbes defines "signs" as just such a device, and further distinguishes between two types of sign, "natural" and "conventional." Natural signs are phenomena in nature that signify a particular occurrence (such as a dark cloud signifying impending rain) and are validated by common experience. Conventional signs, however, are "those which are applied of our own accord," whose meanings are not derived from experience of the 'natural' order of things, but rather from agreed upon convention (such as a dove symbolizing peace).[17] Among conventional signs Hobbes includes "human vocal sounds [*voces humanae*] connected in a certain way for signifying the thoughts and motions of the mind," and he concludes that "the difference between a mark and a sign, therefore, is that the former is instituted for our own sake, the latter for the sake of others."[18]

The general label Hobbes applies to marks and signs in language is "names," which perform the job of both marks and signs. Names serve as conventional signs for the communication of ideas, but "they perform the function of marks before that of signs," because they are first applied in the mind to aid the memory of a particular sensation of concept in the under standing.[19] In this way, Hobbes's theory of language has an empirical basis: names have their origins in sensation, being the tokens of memory which itself is "decaying sense."[20] The names we give to things are rooted in the mnemonic process, and memory is founded on the perception of individual objects of experience. Furthermore, Hobbes's linguistic philosophy is explicitly nominalistic, since "the things named, are every one of them Individuall and Singular."[21]

According to Hobbes (like Gassendi), names do not signify objects themselves, but rather concepts of objects perceived: "the conceptions answering to those things in our mind are the images and phantasms of individual animals or other things."[22] The 'universal' character of certain names ('stone,' 'horse') does not come from nature. Hobbes criticizes those who suggest otherwise, asking "who can come to think that the natures of things display themselves in their names," and concludes that "the dispute over whether names signify matter, form, or a composite of them and other disputes of this kind are characteristic of erring metaphysicians who do not understand the words about which they are arguing."[23] The process by which humans use individual names to signify general categories of experience is an act of transference or translation.[24] In other words, the universalizing function in language is a kind of shorthand, a means of representing objects not as they actually are in nature, but in a *constructed* sense, saving the mind the impossible task of generating separate marks for every individual encountered. Thus, *pace* Aristotle and scholastic grammarians, "'genus' and 'species' are not correctly used in metaphysics for things, and 'definition' for the nature of a thing, since they are only the significations of our conceptions of the nature of things."[25]

Hobbes's theory of truth is also nominalistic and constructivist. Like Gassendi, Hobbes suggests that truth and falsity are based in language, rather than in objects: "For *True* and *False* are attributes of Speech, not of Things," and "*truth* consisteth in the right ordering of names in our affirmations."[26] The terms 'true' and 'false' therefore refer to agreements between conventionally assigned terms, and not to essences or to a natural order. Conventional definitions are related to sense perceptions, but knowledge of truth is not derived from the senses. As Hobbes contends, "names are not established by the essences of things but according to the will of men. For this reason it happens that whoever departs from the agreed appellations of things is deceived not by things nor by sense experience."[27] In language reminiscent of Gassendi's, Hobbes establishes a definition of truth that is explicitly linguistic: "The words 'true,' 'truth,' and 'true proposition' have the same force."[28] This strategy allowed Gassendi to mitigate his skepticism about human understanding, and for Hobbes it promises a measure of certainty since, as Jesseph notes, the notion of linguistic truth has "the virtue of making such knowledge completely certain: if demonstrations are grounded in the arbitrary imposition of words and confined to the analysis of names . . . there is no danger that our reasoning might lead to falsehood."[29]

It is on this subject that Hobbes directly challenged Descartes in his set of objections to the *Meditations*.[30] These objections were contributed in 1641, and are therefore an indication that Hobbes's philosophy of mind was well developed several years prior to the publication of *Leviathan* and *De corpore*. Hobbes was especially critical of Descartes' belief that the mind is able to directly capture the essence of an object, and he insists that reasoning is based on language (mental or spoken) and the senses, since "reasoning will depend on names, names will depend on the imagination, and imagination will depend (as I believe it does) merely on the motions of our bodily organs."[31] Whereas Descartes believes the mind is capable of innately demonstrating the 'existence' of objects in the world, Hobbes's definition of existence is conventional and linguistic: it is the insertion of the *copula*, or the verb 'is,' between two terms of a statement that defines existence. According to Hobbes:

> if it turns out that reasoning is simply the joining together and linking of names or labels by means of the verb 'is' . . . It would follow that the inference in our reasoning tell us nothing about the nature of things, but merely about the labels applied to them; that is, all we can infer is whether or not we are combining the names of things in accordance with the arbitrary conventions which we have laid down in respect of their meaning.[32]

If, as Hobbes goes on to claim, "essence, in so far as it is distinct from existence, is nothing more than a linking of terms by means of the term 'is,'" then there is not only no basis for assuming Descartes' central claim in the *cogito*, but also no foundation for the assertion that real universals exist and may be perceived through pure cognition.[33]

As Hobbes sees it, language or perhaps more accurately 'linguistic capacity' is more than simply the ability to join names together at random. His vision for natural philosophy is that logic should be applied to sense experience in order to investigate the causes of phenomena. The method of proper scientific investigation involves both 'analysis' and 'synthesis,' which he defines, respectively, as "ratiocination from the supposed construction or generation of a thing to the efficient cause or coefficient causes of that which is constructed or generated," and "ratiocination from the first causes of the construction, continued through all the middle causes till we have come to the thing itself which is constructed or generated."[34] Put more simply, analysis is reasoning from observed effects to determine causes, while synthesis reasons from known causes to effects.

Hobbes believes that analysis is by far the more common and reliable tool for pursuing the natural sciences, since we are not naturally privileged with innate awareness of the causes of natural phenomena. This follows his epistemology of language: we observe singular objects in nature, and the mind combines the marks of those objects to form a more general, composite understanding. Hobbes maintains that "all singulars are composed from the causes of universals or simples," and we may reconstruct those causes "by resolution," which is a kind of mental computation or manipulation of marks and names to form propositions. Famously, Hobbes characterizes this 'mental accounting' as a kind of 'computation.' As he explains in *Leviathan*:

> *Subject to Names*, is whatsoever can enter into, or be considered in an account; and be added one to another to make a summe; or substracted one from another, and leave a remainder. The Latines called Accounts of mony *Rationes*, and accounting *Ratiocinatio*: and that which we in bills or books call *Items*, they called *Nomina*; that is, *Names*: and thence it seems to proceed, that they extended the word *Ratio*, to the faculty of Reckoning in all other things. The Greeks have but one word, for both *Speech* and *Reason*; not that they thought there was no Speech without Reason; but no Reasoning without Speech: And the act of reasoning they called *Syllogisme*; which signifieth summing up of the consequences of one saying to another. And because the same things may enter into account for divers accidents; their names are (to shew diversity) diversely wrested, and diversified.[35]

By tracing the etymology of the terms 'name' and 'rational' (from which Hobbes's favored term for reasoning, 'ratiocination' is derived), Hobbes presents a picture of reasoning as literally 'reckoning,' or accounting with names, an explicitly computational operation. Hobbes develops this conception in *De corpore* as well, where he reduces all reasoning to mental addition and subtraction and describes the method by which individual thoughts or conceptions are compounded to form more complicated ideas or statements.[36] In a sense, Hobbes is proposing that reasoning makes use of a mental 'calculus' of concepts, and that

> These operations are not incident to Numbers onely, but to all manner of things that can be added together, and taken one out of another. For as the Arithmeticians teach to adde and substract in *numbers*; so the Geometricians teach the same in *lines*, *figures* (solid and superficiall), *angles*, *proportions* . . . The Logicians teach the same in *Consequences of words*; adding together *two Names*, to make an *Affirmation*; and *two Affirmations*, to make a *Syllogisme*; and *many Syllogismes* to make a *Demonstration*; and from the *summe*, or *Conclusion* of a *Syllogisme*, they substract one *Proposition*, to finde the other.[37]

This process, Hobbes explains, is the basis for all philosophy, and it explicitly informs his approach to mathematics. Insofar as Hobbes's philosophy of language is 'mathematical,' as we will see, his philosophy of mathematics is equally 'linguistic.'

Hobbes's Philosophy of Mathematics

Like Isaac Barrow and Newton after him, Hobbes subscribed to what can be best termed a 'mechanical geometry' that is founded on multiple levels of constructivism. Motion is the basis for Hobbes's materialist geometry and is the foundation for analytical methods in mathematics, since Hobbes originally conceived his new geometry as "analysis by motions or the method of motion."[38] To this extent, Hobbes's philosophy of geometry may be described as 'mechanical constructivism':

> The paths of motions simpliciter (in which geometry consists) ought to be investigated in the first place, and then the paths of motions generated and obvious, and finally the paths of motion internal and invisible (which physicists study). Thus those who study natural philosophy study in vain unless they take their principles of inquiry from geometry; and those who write or lecture about such things and are ignorant of geometry abuse their readers and listeners.[39]

Whereas Descartes, for example, believed geometry to be the foundation of physics because it offered insight into nature beyond what was perceivable by the senses and expressible in mere language, Hobbes's geometry was rather an extension of his general, nominalistic and empiricist philosophy of representation, which involved mathematical and epistemological constructivism as well. For Descartes, geometry was generalizable because geometrical relations were installed by the creator, and because its universal truths are directly perceived by the mind. Hobbes, on the other hand, allowed geometrical definitions and statements to have general force by the same principle that made statements in language general: as human constructions they are based on experiences of individual phenomena, and are not eternal truths abstracted from the essences of things.[40]

Hobbes conceived the objects of geometry to be representations of the motions of certain basic objects. As he notes in *De Corpore*, "*Lines*, *superfices*, and *solids*, are exposed, first, by *motion* . . . but so as the marks of such motion be permanent; as when they are designed upon some matter, as a line upon paper; or graven in some durable matter."[41] A line, in other words, is the path traced by a point in motion; a plane (or 'superfice') is the surface swept out by a moving line, etc. A geometrical representation, therefore, is not the abstracted essence of an object or a Platonic form, but rather a kind of 'time exposure' tracing the path of a geometrical object or objects in motion over individual moments. This is not necessarily to say that Hobbes imagined geometrical shapes to be literally 'in motion'; rather, he treated geometrical objects as if they represented motions, because he recognized geometry as valid only insofar as it described actual physical phenomena. Geometry, then, is a language for expressing physical relationships. Since Hobbes's ontology was mechanical and since he denied the existence of 'abstract essences' which could be separated from matter geometry must be a 'mechanical' science in order to properly represent nature.

Hobbes's mechanical physics also relies on two important concepts: 'endeavor' (*conatus*) and 'impetus,' which are terms that apply to physical moving bodies and also to the construction of geometrical figures. Hobbes defines Endeavor as "motion made in less space and time than can be given . . . that is, motion made through the length of a point, and in an instant of time" or, an instantaneous motion across an extremely tiny distance. Likewise impetus is defined as "quickness of motion," or

> the swiftness or velocity of the body moved, but considered in the several points of time in which it is moved. In which sense *impetus* is nothing else but the quantity or velocity of endeavour. But considered with the whole time, it is the whole velocity of the body moved taken together throughout all the time, and equal to the product of a line representing the time, multiplied into a line representing the arithmetically mean *impetus* or quickness.[42]

If endeavor is the instantaneous motion of a point (which can be located on an extended magnitude) over a very short distance, impetus is the velocity of the point. This helps to explain the generation of geometrical figures because, as Jesseph explains, we can imagine a figure a curve, for instance as being generated by the motion of a point, and the sweep of the curve can be represented by individual stages of that point's motion. Each stage has a particular instantaneous direction and velocity, and each stage is also an indefinitely small portion of the curve, where the sum of all of the motions will add up to the curve itself.[43] This approach to geometry is quite similar to Newton's calculus of fluxions and, as Jesseph also suggests, may have been derived in part from Galileo's mechanics in *Two New Sciences*.

In many respects, however, the similarity between Hobbes's method of geometrical analysis and later analytic approaches is misleading. Quite simply, Hobbes vehemently rejected analytic geometry and symbolic algebra, which were being developed into powerful tools by contemporaries like Descartes and John Wallis. In particular, Hobbes's nominalism played a role in his disdain for algebra, which he describes as

> the *brachygraphy* of the analytics . . . an art neither of teaching nor learning geometry, but of registering with brevity and celerity the inventions of geometricians. For though it be easy to discourse by

> symbols of very remote propositions; yet whether such discourse deserve to be thought very profitable, when it is made without any ideas of the things themselves, I know not.[44]

This passage reflects many of Hobbes's concerns about the new methods in mathematics and illuminates the central issues in his quarrel with Wallis. Its three major points can be summarized as follows: (1) traditional geometry, by means of syllogistic reasoning, allows for the discovery of hidden causes in the figures being analyzed; (2) algebra, which arithmetizes the spatial relations in geometry, is at best a kind of short hand (brachygraphy) for geometry, useful perhaps for taking notes but not for teaching or discovering; and (3) symbolic analysis is too far removed from the true objects of geometry for its 'marks' to accurately stand for the concepts being signified.

Beginning with the last point first, it is clear that Hobbes's criticism of algebraic notation stems largely from his general philosophy of representation.[45] Recalling Hobbes's theory of language, physical marks used in communication are drawn from the memory, which is itself derived from an imagination formed by sense experience a written word is a sign of a mental sign, which is the sign of a thing. Since thinking involves the retention of mental images based on sensations, Hobbes believed that actual physical diagrams were the closest approximation of actual experience. This shows his concern about the ontological content of the entities the symbols purport to represent. In Hobbes's scheme, there is a very real concern with the decay of information through repeated 'translations' from experience to concepts, and his positive approach to science involves devising methods to minimize this loss of information by fixing precise definitions and significations onto words used. Though he believed the true essence of a thing would never be captured, Hobbes hoped nonetheless that through precise conventional definitions humans might have a greater understanding of what they are talking about when they describe nature. In the case of analytic geometry, however, there is an extra translation made that Hobbes sees as unnecessary and damaging:

> For the conception of the lines and figures (without which a man learneth nothing) must proceed from words either spoken or thought upon. So that there is a double labour of the mind, one to reduce your symbols to words, which are also symbols, another to attend to the ideas which they signify.[46]

Since geometrical figures are already translations of experience into symbols, he finds no reason to take the step of an additional translation into further symbols, which only makes their original signification more uncertain. It is the principle of parsimony that drives this critique, but it is drawn from his general theory of meaning.

Hobbes was not, however, claiming that symbolic notation is entirely useless. He considered all reasoning to be in a sense symbolic, since marks are symbols (or signs) of concepts, and believed there was no possibility of reasoning on actual 'things' (as there was for Descartes). Rather, the issue was again to find the most accurate and secure symbolic techniques, and to this end he did allow that symbolic shorthand is acceptable for the experienced geometrician to briefly note his discoveries to himself. In *Six Lessons to the Professors of Mathematics*, though, Hobbes makes it clear that he thinks symbolic geometry is not fit for public demonstrations: "Symbols are poor unhandsome, though necessary, scaffolds of demonstration; and ought no more to appear in public, than the most deformed necessary business which you do in your chambers."[47] Hobbes's perhaps excessive rhetoric must be understood here in the context of his extremely acrimonious debate with Wallis, to whom he was addressing that passage, but the point nonetheless is that symbols place an unnecessary burden on communication. Since the most essential acts of communication in geometry are presenting persuasive demonstrations and instructing students, Hobbes felt that analytic geometry was far inferior to traditional methods because traditional methods were easily comprehended by all. Hobbes goes so far as to suggest that nothing new can be discovered through analytic geometry, stating that Wallis "mistook the study of *symbols* for the study of *geometry*, and thought symbolical writing to be a new kind of method, and other men's demonstrations set down in symbols new demonstrations."[48]

The question remaining, then, is why Hobbes thought 'traditional' geometry was more secure than algebra and analytic geometry. Despite questions about deficiencies in his technical abilities, the epistemological principles in Hobbes's mathematical philosophy are consistent with his overall treatment of language and signification, so I suggest he should be taken seriously as a philosopher of mathematics, if not always as a mathematician. We have already seen that Hobbes's philosophy of mind privileges not the mind's inherent capacity to penetrate nature, but rather the faculty of the understanding to construct knowledge based on the marks and signs drawn from experience. Certainty comes only when the objects being discussed are conventionally defined, since then their precise meanings may be known. 'Certainty' is thus distinguished from 'ontological reality.'

This scheme also applies to geometry, as Hobbes explains in the dedication to *Six Lessons*:

> the science of every subject is derived from a precognition of the causes, generation, and construction of the same; and consequently where the causes are known, there is place for demonstration, but not where the causes are to seek for. Geometry therefore is demonstrable, for the lines and figures from which we reason are drawn and described by ourselves.[49]

Because of his nominalism, Hobbes believes that all general names and definitions are conventional, so truth can only be assigned insofar as statements made about general classes agree with the recognized definitions applied to those objects. For Hobbes, geometry is entirely dependent as a science on the application of conventional definitions:

> as they, who handle any one part of geometry, determine by definition the signification of every word . . . so must he which intendeth to write a whole body of geometry, define and determine the meaning of whatsoever word belongeth to the whole science.[50]

Geometry has certainty precisely because it uses axioms and definitions that are defined by humans, but that certainty applies only insofar as its universal objects like geometrical figures are 'constructed.' As Jesseph notes, according to Hobbes, such certainty is possible only in pure mathematics, since the physical world does not present causes to our understanding. Thus, geometry is 'maker's knowledge,' since its certainty is grounded "in our construction of the objects known."[51] Hobbes's philosophy can, therefore, be seen to be underwritten not only by mechanical and mathematical constructivism, but also by a general epistemological constructivism that much like Gassendi's is drawn from his commitment to nominalism.

Hobbes therefore endorsed a version of mathematical constructivism, which (like Gassendi) he derives from nominalism. This, however, raises a fascinating apparent paradox in Hobbes's thought: since Hobbes holds that truth is essentially linguistic, and being a nominalist he holds to a conventional explanation of general classes and concepts, why does he reject analytic geometry, which is even more explicitly linguistic and conventional than traditional Euclidean methods? Part of the answer lies in the fact that Hobbes felt that algebra, as Jesseph notes, 'distracts' mathematicians' attention from "contemplation of magnitudes," since as a rule it involves extra steps in translation, and prevents cause effect demonstrations.[52] One may, for instance, assign X and Y to whatever variables one wishes and speak of them as if they were marks for particular things, but as Hobbes states in *De corpore*, "the use of names is private and arbitrary," and using a mathematics based entirely on names that are so private and arbitrary that their signification changes from equation to equation prevents causal understanding based on demonstration from definition. In Hobbes's eyes, Wallis and his colleagues are attempting to acquire universal (real) knowledge from this procedure, which is doubly problematic: in the first place, such knowledge is inherently unattainable, and in the second, what general knowledge *is* available is only valid when grounded in clear definition, which algebra appears to lack. The irony here is that in some sense Hobbes, the 'ultra nominalist,' is unwilling to follow his nominalism to the ultimate conclusion, which would make mathematics a kind of arbitrary formalism.

Helena Pycior even claims that "despite his animadversion toward early modern algebra, [Hobbes's] nominalist approach potentially nudged algebra in the symbolical direction."[53] As the final chapter will discuss, this move from nominalism to formalism is exactly the path taken by Berkeley.

Locke

There are very good reasons for considering John Locke's empiricist reconstruction of epistemology in this book: First, Locke had read and admired Gassendi's work, which likely influenced his thought to some extent. Second, Locke's empiricism was strongly tinged with nominalism, and his treatment of mathematics (such as it is) has a significant nominalist basis. Third, Locke was indisputably part of the same intellectual milieu as Newton, with whom he had a personal friendship and whose system of natural philosophy he sought to emulate in his ethics and political philosophy. But Locke is a complex figure, and I will not attempt to argue he was a thoroughgoing nominalist like Gassendi, Hobbes, or Berkeley. Rather, I will briefly discuss how Locke's general epistemology dealt with mathematics, where his positions were in the 'spirit' of nominalism if not explicitly nominalist in themselves. The chapter will then conclude with a more thorough examination of George Berkeley, another 'British empiricist' whose philosophy is more comfortably characterized as nominalist and constructivist.

The relationship between Locke and Gassendi has been treated in several recent works on Locke, and there has been some controversy concerning the extent of Gassendi's influence on the development of his thought. David Fate Norton first made a strong claim for a connection between Locke and Gassendi, when he argued both for Gassendi's considerable influence on Locke and also for Gassendi's replacement of Locke as the "founder" of modern empiricism.[54] This argument prompted responses from a number of scholars, many of whom agree in principle with Norton but differ somewhat over the details of his interpretation.

Richard Kroll, for example, criticized Norton for relying on only "morphological" similarities between the writings of Gassendi and Locke to connect their views. Kroll drew attention to the need to establish direct, textual evidence of a link between the two philosophers (which Kroll maintained Norton failed to do), and he determined several possible Gassendist influences on Locke: Bernier's *Abrégé de la Philosophie de Gassendi* (which Locke owned), Walter Charleton's epitome of Gassendi's *Animadversiones*, Robert Boyle's writings, Thomas Stanley's *History of Philosophy*, and of course Gassendi's own works, the *Animadversiones* and the *Syntagma.*[55] Bernier is ruled out because his *Abrégé* was not published until 1674, by which date "the most substantial aspects of [Locke's] *Essay* were already in draft," and could not have significantly influenced Locke.[56] Charleton is not given much discussion, perhaps because there is no evidence that Locke ever owned a copy of his work. Boyle’s writings are given due consideration, but while Kroll does not discount their impact on Locke, he does question the influence of Gassendi on Boyle, who claimed to have had little acquaintance with Gassendi’s work.[57] Finally, the more obvious and direct hypothesis that Locke read Gassendi in the original is also rejected, based on the grounds that Locke would not have wanted to plow through Gassendi’s “turgid Latin.”[58] The remaining choice is that Locke read Stanley’s *History of Philosophy*, which, according to Kroll, is notable for containing a translation “of [Gassendi’s] entire *Syntagma*“ in its section on Epicurus.[59] Kroll notes with amazement that Stanley’s translation appeared in 1659, only a year after the publication of Gassendi’s *Opera* (where the *Syntagma Philosophicum* first appeared), and speculates that this accounts for Locke’s thorough knowledge of Gassendi’s “canons” of logic. The textual link having been discovered, Kroll now justifies morphological comparisons between Locke’s work and Gassendi’s.

There are considerable problems with Kroll’s analysis, some of which are addressed by Fred Michael and Emily Michael in an essay published a few years after Kroll’s.[60] The most significant error lies in Kroll’s confusion about the content of the ‘*Syntagma*‘ he claims was translated by Stanley. In his essay, Kroll cites Gassendi’s “posthumous *Syntagma Philosophicum* (1658),” which was included in Gassendi’s *Opera*, as the definitive expression of Gassendi’s Epicurean views, and proceeds throughout the rest of his essay to refer to this text as the *Syntagma* of Gassendi. Stanley did indeed produce a translation of *a* ‘*Syntagma*‘ written by Gassendi, but unfortunately for Kroll the *Syntagma* in question was not the compendious *Syntagma philisophicum* of the *Opera*, but rather Gassendi’s earlier *Epicurean Syntagma*, which was appended to the *Animadversiones* in 1649.[61] The *Epicurean Syntagma* was, in fact, a much shorter outline merely of Epicurus’ philosophy, and does not contain Gassendi’s own commentary and emendations. Stanley’s translation of Gassendi’s *Epicurean syntagma*, therefore, could have introduced Locke to the ideas of Epicurus in a general way (which he also could have accomplished easily from a number of other sources), but does not constitute the decisive link between Locke and Gassendi that Kroll claims.

As Michael and Michael argue, however, there are other significant reasons for connecting Gassendi with Locke. Granting that there are ‘morphological’ similarities between the two philosophies (which I will discuss), the question is one of historical causality: how could Locke have encountered Gassendi’s opinions, and what is the likelihood that they affected the composition of the *Essay*? In the first place, one source Kroll overlooks is Gassendi’s objections to Descartes’ *Meditations*, which as I argued in the previous chapter was a widely available introduction to Gassendi’s epistemology in the mid seventeenth century. Locke possessed a copy of this work in 1681, and it is possible that he had consulted it prior to that date. Michael and Michael also call Boyle’s ignorance of Gassendi into question, citing a letter in which Boyle refers to Gassendi as a "great favorite" of his.[62] Finally, Michael and Michael question the logic behind Kroll's assertion that "turgid Latin" would have prevented Locke from reading Gassendi in the original, a criticism I wholeheartedly second.[63]

Beyond these arguments, it must be left open whether Gassendi directly influenced Locke's epistemology. Clearly, Locke had some familiarity with Gassendi's work and he was influenced by Gassendist ideas in indirect ways as well by having read Boyle, through his friendship with Bernier, by association with the early members of the Royal Society but I hesitate to make any specific claims for direct influence with certainty. Given the fact that Locke's epistemology resembles Gassendi's in many crucial ways, and given Locke's circle of correspondents and scientific friends, I find no problem in placing Locke within the intellectual *milieu* of Gassendi and the nominalists. Locke was also familiar with Hobbes's philosophy, and though the two differed on matters of political theory, Locke owes Hobbes a clear debt.[64] Locke was motivated by many of the same concerns as Gassendi and Hobbes, and his epistemology is (as will be shown) fairly consistent with nominalism. He has also been interpreted, both by his contemporaries and by modern scholars, as continuing a tradition of empiricism that began with Gassendi and carried through the work of Berkeley and Hume that opposed the rationalistic philosophy of Descartes and Leibniz.

In the context of my argument it is Locke's epistemology that is important, and specifically his analysis of language, which maintains a conventionalist and nominalist approach similar to Gassendi's and Hobbes's. Locke was not a mathematician but was conversant in natural philosophy and friendly with Boyle and Newton, and his analysis of concepts in language led him to a consideration of the method of geometry and arithmetic. This connection is revealing. I will first briefly sketch Locke's approach to signification in language and mental operations, and then summarize his attitude towards mathematics and its relationship to his overall theory of representation.

The Essay and Epistemology

In Book 1 of the *Essay*, Locke famously rejects the concept of 'innate notions.' His purpose is to inquire into the possibility of attaining certain knowledge, and Locke wonders whether our knowledge is determined by ideas imprinted on the mind at birth. Although Locke apologized for resorting so often to a term that is frequently and imprecisely used 'idea' he explained that it is

> [a] term, which, I think, serves best to stand for whatsoever is the Object of the Understanding when a man thinks, I have used it to express whatever is meant by *Phantasm*, *Notion*, *Species*, or whatever it is, which the Mind can be employ'd about in thinking.[65]

Ideas, however, are established by experience, and according to Locke there are no "Characters, as it were stamped upon the Mind of Men" or "Original Notions or Principles" that guide the understanding.[66] Rather, "The Senses at first let in particular *Ideas*, and furnish the yet empty cabinet"; the mind at birth is a *tabula rasa* that over time collects an inventory of ideas in its memory, from which words and eventually language are abstracted.[67] Locke is quite clear that it is solely "From *Experience*: in that, all our Knowledge is founded," and echoing Hobbes that the root of experience is sensation of things around us: "*Ideas* in the understanding are coeval with *Sensation*."[68] This is both a clear refutation of the Platonic doctrine of 'recollection,' as well as a contradiction of Descartes' assertion in the *Meditations* that knowledge originates in the intellect alone.

While Locke states that ideas are representations, they do not directly represent the essences of the things experienced. We perceive the qualities of sensible objects, and from these the mind 'composes' or puts together a complex picture of the object or phenomenon sensed.[69] An idea is something in the mind that stands for a particular experience, but "we *may not* think (as perhaps is usually done) that they are exactly the Images and *Resemblances* of something inherent in the subject."[70] Like Gassendi and Hobbes, Locke recognizes the futility of tying individual names to each singular experience, so instead the mind generalizes from experience. This process "is called ABSTRACTION, whereby *Ideas* taken from particular Beings, become general Representatives of all of the same kind; and their Names general Names, applicable to whatever exists conformable to such abstract *Ideas*."[71] In other words, we do form generalized ideas of things, but those generalizations are constructed by the mind in order to conform particular experience to secure tokens in the memory. Like his nominalist colleagues Locke saw words and abstract generalizations as artificial. Since we can only construct these complex categories by combining our experiences of individual substances and since experience itself is limited to the 'exterior' nature of a thing we can "have no Knowledge beyond that ['superficial ideas'], much less of the internal Constitution, and true Nature of things, being destitute of Faculties to attain it."[72]

The resulting epistemological position is again, like Gassendi's and Hobbes's that truth is a notion to be applied only to propositions. Locke goes even further to suggest that truth is not simply a matter of spoken propositions, but rather that *all* thought involves "secret or tacit Proposition," since

> our *Ideas*, being nothing but bare Appearances or Perceptions in our Minds, cannot properly and simply in themselves be said to be *true* or *false*, no more than a single Name of any thing, can be said to be *true* or *false*.[73]

This process is essentially constructive, since it involves joining two or more distinct impressions of individual substances to produce complex ideas, which are themselves propositions. As Michael Ayers explains:

> In the geometrical context the issue between imagism and intellectualism took the form of a dispute about the role of diagrams. On the Cartesian view, diagrams, actual or imagined, at best prompt the intellect to form or make explicit its purely mental conceptions. For imageists like Locke, however, actual or imagined diagrams are themselves the essential objects of geometrical reasoning.[74]

Locke, in fact, devotes an entire book in the *Essay* to the examination of words, and he laments the fact that people often suppose that words signify actual objects in “the reality of things.” Instead, he maintains that it is “*by a perfectly arbitrary Imposition*“ that words signify anything at all, and he joins Gassendi in rejecting the notion that categories in the mind (the basis for words) constitute *real* universals.[75] Here Locke affirms the nominalist position that things are particulars, but also restates his theory of abstraction, according to which words are made general “by separating from them the circumstances of Time, and Place, and any other *Ideas*, that may determine them to this or that particular existence.” In this fashion, words are made “capable of representing more Individuals than one,” but they do not directly represent ontologically real categories, since “*General and Universal*, belong not to the real existence of Things; *but are the Inventions and Creatures of the Understanding . . . and concern only Signs, whether* Words or *Ideas*.”[76] When people speak of ‘essences’ and ‘species,’ they are speaking not about an inner nature of things but rather “nothing else but an Artifice of the Understanding.”[77] We cannot know the “real Essences” of things, only the “nominal Essences,” which “’tis evident they *are made by the Mind*, and not by Nature.”[78] This last statement resonates strongly with the nominalist/constructivist theory: general knowledge about nature is possible, but insofar only as it is constructed (by the mind) out of discrete sensory experiences.

One area of knowledge that Locke discusses with enthusiasm is mathematics, and it remains to discuss how his views on this subject relate to his overall epistemology. Locke believes that mathematical concepts like any concepts are not innate, and that we must obtain experiences of quantity before numbers become meaningful.[79] He also (unlike Hobbes) expresses a clear preference for arithmetic over geometry. Here Locke points out that in certain cultures (such as the ‘natives’ of America) there is neither a concept of general propositions in mathematics nor a need for them:

> They are the Language and Business of the Schools, and Academies of learned Nations, accustomed to that sort of Conversation, or Learning,

> where Disputes are Frequent: These Maxims being suited to artificial Argumentation . . . are not much conducing to the discovery of Truth, or advancement of Knowledge.[80]

What Locke is particularly targeting here is the notion as old as geometry itself that certain 'maxims' or axioms are self evident and necessarily true.

The basic idea of 'number,' on the other hand, is one of the most easily 'suggested' ideas to our minds, and one of the first we obtain. Locke considers number, or "unity," to be a "simple mode" and argues that names or marks serve to signify combinations of unities. In other words, the building block of number is the concept of unity, and 'a number' (for instance, '10') is 'unity + unity + unity . . . etc'; the number that signifies a combination of unities is an arbitrary mark serving to simplify expression.[81] In order to reason with numbers, "the mind distinguish carefully two *Ideas*, which are different one from another only by the addition or subtraction of one Unitie . . . [and] retain in Memory the Names, or Marks, of the several Combinations from an Unitie to that Number."[82] Unity is a simple idea, drawn from experience of quantity, while 'numbers' are combinations of ideas, or 'modes.' Therefore it can safely be concluded that Locke's view on the ontological status of number is consistent with his view of other ideas: an idea can only reflect the exterior of a thing, and combinations of ideas involve tacit mental propositions. Reasoning with numbers, therefore, is no different than reasoning with words both are conventional and to some degree constructed.

Locke's view of geometry reflects his general epistemology, but it is slightly more complicated than his approach to numbers. He defends the use of abstract geometrical ideas, but denies that they must refer to ontologically real objects:

> For were there no Circle existing any where in the World . . . yet the Idea annexed to that Name would not cease to be what it is; nor cease to be as a pattern, to determine which of the particular Figures we meet with, have, or have not a Right to the Name Circle.[83]

This position supports Locke's general theory of reference: he relates the discussion of geometry to the use of compound words like 'unicorn' or 'mermaid,' which are concepts that signify an abstract idea that is true or false only when used in a proposition. Like Hobbes, Locke believed geometrical terms such as 'circle' can be used correctly only insofar as they fit the conventional definition applied to 'circle' (just as a mermaid need not exist in reality in order for there to be true propositions regarding 'mermaids'). General ideas and this is a crucial point need not refer to physically real objects, since by definition general objects are constructed by the mind (and are not 'natural').

Knowledge, such as it is, is founded upon the relations between ideas in the mind and on the consistency of those relations: Locke calls "intuitive Knowledge" more certain than "demonstrative Knowledge," and cites mathematics as the paradigm for the latter, but he in fact seems torn between a desire to hold up mathematics as a privileged mode of discourse and a realization that by his own rules it cannot be so.[84] In the end, he seems to cast mathematics into the same lot as natural language:

> I doubt not but that it will be easily granted, that the *Knowledge* we have *of Mathematical Truths*, *is* not only certain, but *real Knowledge*, and not the empty vision of vain insignificant *Chimeras* of the Brain: And yet, if we will consider, we shall find that it is only of our own *Ideas*. The Mathematician considers the Truth and Properties belonging to a Rectangle, or Circle, only as they are *Ideas* in his own mind.[85]

That is not to say that Locke dismisses mathematical knowledge; indeed, he finds the 'language' of mathematics to be better suited for enunciating clear propositions than natural languages. But nor does he exempt mathematics from the rules that govern all languages, and propositions made in language. As Locke concludes:

> All the Discourses of the Mathematicians about the squaring of a circle, conick Sections, or any other part of Mathematics, *Concern not* the *Existence* of any of those Figures: but their Demonstrations, which depend on their *Ideas*, are the same, whether there be any Square or Circle existing in the world, or no.[86]

Mathematics in this case geometry is a language that deals exclusively with general ideas. As such, it is capable of producing 'true' statements that nonetheless describe abstract or 'fictional' entities. This type of constructivist argument is familiar from the medieval nominalist tradition, and will appear again in explicitly mathematical terms in the next chapter's discussion of Isaac Barrow's constructivist philosophy of geometry.

Berkeley

First, however, it remains to consider the philosophy of George Berkeley, who was himself an enigmatic and iconoclastic figure in the empirical tradition of British philosophy. I will argue that Berkeley does not merely stand out as an exemplary member of the nominalist tradition, but that he embodies the culmination of that approach to questions concerning language and mathematics, most concretely articulating a nominalist and conventionalist theory of signification and a constructivist philosophy of mathematics. These beliefs have a clear lineage back to Gassendi, but Berkeley's mathematical sophistication allows him to develop a philosophically rigorous and ultimately important nominalist philosophy of mathematics that was more relevant to the concerns of practicing mathematicians of his day than any of his predecessors'. In fact, some historians have argued that Berkeley helped to define the character of British mathematics for a number of years.[87]

Apart from the internal similarities to Gassendi and others there is ample reason to consider Berkeley's philosophy within the context of the nominalist tradition I have been describing. Helena Pycior writes that Berkeley, "seems to have read widely during his late undergraduate and immediate postgraduate years," commenting that his mathematical interests led him to "read Barrow, Wallis, and Newton and perhaps Hobbes as well."[88] While Berkeley does not often mention other philosophers by name in his later writings (with the notable exceptions of Locke and Newton), in his early *Commonplace Book*, written between 1705 and 1708, he makes frequent reference to the writings of all the above authors. Interestingly enough, Berkeley's criticisms of Hobbes are often harshest, while his comments on Locke are, if sometimes critical, also measured and respectful.[89] Despite his criticism of Locke's theory of ideas, Berkeley praises Locke as the writer "who, if anyone did, knew well all the defects of the human intellect and their remedies, [and who] recommends, as infinitely useful, the study of mathematics in general."[90] Locke was, in other words, a seminal influence on Berkeley, and Berkeley's later criticism of Locke is probably explained by this fact.[91]

What is more striking, however, are Berkeley's references to mathematicians of his day, including John Wallis and, especially, Isaac Barrow, whom he appears to have found influential not just for his mathematical philosophy, but also for his general theory of representation. Berkeley seems to have studied Wallis' *Mathesis Universalis*, to which he refers approvingly in the *Mathematical Miscellanies*, and as I will argue later, his formalistic reading of algebra is similar to Wallis' own approach. Of greater significance, though, are the far more frequent references to Barrow in Berkeley's *Commonplace Book*, which demonstrate a familiarity and sympathy with Barrow's *Lectiones Geometricae* and *Optical Lectures* (and perhaps with the *Mathematical Lectures* as well), which, as the next chapter will argue, had a major role in developing a constructivist theory of geometry.[92] As Pycior notes, "Barrow's view of number as a 'note' or 'sign' . . . became one of his [Berkeley's] stock examples against abstraction," and Berkeley's "early acceptance of symbolical reasoning and Barrow's view of number . . . contributed toward his innovative theory of language."[93] This is evidence that Berkeley conceived his general philosophy of representation and his philosophy of mathematics simultaneously, and that his reflections on language and mathematics were mutually reinforcing. Berkeley's strongly conventionalist and nominalist reading of language was intimately connected to his theory of mathematics as a science of signs, and his view of the denotational content of mathematical symbols is entirely consistent with his interpretation of language in general.

'Esse est percepti': The 'Principles of Human Knowledge' and Berkeley's Philosophy of Mind

One of the major problems that concerned philosophers over the course of the seventeenth century was explaining the link between sensory perception and the formation of ideas in the mind. As we have seen, the strategy of nominalists from Ockham through Gassendi and Hobbes was to assert that we may only access information about the physical world through the senses, and that our ideas do not correspond in any necessary way to essences or real natures. Berkeley's contribution to this problem was quite original and radical.[94] His proposal denies both rationalist essentialism and Lockean idealism: according to Berkeley, we neither have access to the real natures of objects of experience, nor do we form abstract general ideas based on particular phenomena we observe. Rather, our ability to form general concepts comes entirely from our use of language, which is an arbitrary mechanism for producing 'signs' that have no necessary relation to objects we experience in the world. As a result, he concludes:

> it doth not appear to me that those notions are formed by abstraction in the manner premised [by Locke and others]; universality, so far as I can comprehend not consisting in the absolute, positive nature or conception of any thing, but in the relation it bears to the particulars signified or represented by it: by virtue whereof it is that things, names, or notions, being in their own nature particular, are rendered universal.[95]

This notion is predicated on the assumption that "all those bodies which compose the mighty frame of the world, have not any subsistence without a mind," which leads to Berkeley's controversial doctrine of immaterialism.[96] While Descartes also famously proposed that existence hinges on the awareness of a perceiving mind, the difference between his theory and Berkeley's is striking: Descartes, who privileged reason over experience, overcame his solipsistic skepticism by rationally deducing the existence of material bodies from the analogy between his own mind and the mind of God. For Descartes, material existence can only be proven rationally because the eternal truths of nature are available only to reason (and guaranteed by God); a mind is necessary to prove that other objects exist, but it does not *create* the objects themselves. Berkeley, on the other hand is willing to adopt just such a radical thesis: material objects, as far as human knowledge is concerned, exist only as representations in our mind, and we have no basis for discussing them other than as constructions.

In presenting this conception of perception and cognition, Berkeley simply takes Gassendi's nominalistic scheme to an extreme conclusion: since the only 'objects' we directly experience are mental constructions drawn from our perceptions, it is the representations themselves that are real to us, not the objects to which we presume they refer. Berkeley bases his position on the assumption that it is impossible to conceive any idea not first acquired by the senses, coupled with the notion that the senses perceive only particulars, and not general or universal qualities. By this reasoning, he denies the existence of abstract ideas, which cannot be located in direct experience, since "as it is impossible for me to see or feel anything without an actual sensation of that thing, so is it impossible for me to conceive in my thoughts any sensible thing or object distinct from the sensation of it."[97] In response to the question of how we form common ideas of things drawn from general experience like the idea of a 'tree' that correlate with our experiences of particular objects, Berkeley dismisses the entire problem. He asks:

> What is all this more than framing in your mind certain ideas which you call books and trees, and at the same time omitting to frame the idea of any one that may perceive them? But do not you yourself perceive or think of them all the while? This therefore is nothing to the purpose: it only shows you have the power of imagining or forming ideas in your mind; but it doth not shew that you can conceive it possible, the objects of your thought may exist without the mind: to make out this, it is necessary that you conceive them existing unconceived or unthought of, which is a manifest repugnancy. When we do our utmost to conceive the existence of external bodies, we are all the while only contemplating our own ideas.[98]

Berkeley does not deny that we have general ideas; he simply denies that such general ideas exist separately from a mind that perceives their objects. This theory of knowledge is nominalist because it denies the existence of real universals and privileges individual sense experiences, and constructivist because it holds that objects are 'artificially' constructed in the mind.

The 'New Theory of Vision' and the Visual Language of Nature

Berkeley's general theory of knowledge was grounded in a unique theory of vision, which further highlights the nominalism and constructivism central to his epistemology. In 1709, a year before the more general *Principles* appeared, he published the *Essay Towards a New Theory of Vision*. Because this work was published while the *Principles* were already being composed, we can view the two jointly as a statement of his epistemology: the *New Theory* presents a concrete and specific explanation of the mechanics of cognition, while the *Principles* outlines Berkeley's general philosophy of representation into which the theory of vision is integrated. In the *New Theory*, Berkeley suggests that just as the ideas we form in our minds are essentially mental constructions, so are the visual cues that produce the images in our minds. Because of the mechanics of vision, Berkeley argues, there is no necessary connection between what we perceive and the 'true' form and nature of objects in the physical world. Whereas other nominalists have argued that construction begins in the formation of ideas in the mind, Berkeley argues that it begins earlier, in sense perception itself he is a sort of 'sensory nominalist.'

Using the example of the perception of depth and distance, Berkeley notes that we understand an object to be a particular size only by processing experience through a logical machinery that determines an object's relationship to other visual cues in the field of view. What informs us that a great cathedral a mile distant, for example, and a tree 30 yards away are not the same height (though they otherwise appear to be) is a body of information not directly associated with direct perception of the scene such as our experience of having been in the cathedral on another occasion, knowledge of the time it would take to walk there, general experience of the sizes of trees, etc. "The estimate we make of the distance of objects considerably remote," Berkeley contends, "is rather an act of judgement grounded on experience than of sense."[99] There is, in other words, no native and unmediated perception; seeing is on the same level, epistemologically, as thinking.

In equating perception with judgement, Berkeley is able to reach the more radical position "that, when the mind perceives any idea not immediately and of itself, it must be by the means of some other idea."[100] This means that the idea formed of the cathedral in the distance is not based on pure sensory perception of the object in view, but rather is contingent on a set of further ideas that tell the mind how to interpret it. In identifying a particular pile of bricks, mortar and wood as a discrete object, the mind 'creates' the idea of a cathedral; the cathedral is, in a sense, simply a 'sign,' which itself is composed of other signs that the mind 'reads' to form an idea of its shape, size, and distance. Furthermore, this particular visual sign will not necessarily have the same signification to every viewer or in every context: Berkeley draws an explicit parallel between the subjectivity of perception and the way in which "a word pronounced with certain circumstances, or in a certain context with other words, has not always the same import and signification."[101] Visual perceptions then have no more necessary connection with an objective 'reality' than the words we use to describe them, since "visible objects are only in the mind" and they do not "suggest aught external, whether distance or magnitude, otherwise than by habitual connection, as words do things."[102]

Berkeley's theory of vision is thus a part of his general semiotic approach to nature and bolsters his critique of Locke's notion of abstract ideas. According to Locke, ideas are formed by abstracting sensible qualities from natural objects we perceive, and although Locke doubted that ideas have a necessary correspondence with real objects, in his scheme vision does provide the objective data from which our sketchy approximations are drawn. Berkeley does not grant even that much: ideas whose subjectivity he is at great pains to emphasize are not *based* on visual impressions, they *are* visual impressions, which is why Berkeley does not believe it is possible to have an idea that is not drawn from particular experience. For this reason Berkeley denies that an idea of a physical object can be decomposed into its constituent qualities, because he himself cannot conceive of a quality existing without an object, or of a quality less object.[103] Using this reasoning Berkeley explicitly targets Locke's explanation for the abstract idea of a triangle, which he finds "incomprehensible."[104] Berkeley's immaterialism is supported by his further reduction of all visual images to patterns of light; what we see are "only light and colors," which the mind resolves into ideas of shape and extension.[105]

Ultimately, Berkeley proposes that our visual awareness constitutes a 'language' whose meaning is every bit as conventional as the spoken and written languages humans use to communicate. He contends:

> We can no more argue a visible and tangible square to be of the same species from their being called by the same name, than we can that a tangible square and the monosyllable consisting of six letters whereby it is marked are of the same species, because they are both called by the same name.

In other words, the connection between the visual image of a 'square' and the thing it purports to represent (the real or 'tangible' square) is no more necessary than the connection between the word and the putative 'thing' that is to say, not at all. Although "it is customary to call written words and the things they signify by the same name," this is done purely for the sake of simplicity, and is entirely a matter of convention, since "it had been superfluous and beside the design of language to have given them names distinct from those of the things marked by them."[106] Language, in its conventional aspect, simplifies representation: hence we do not need to say, 'I drew a representation of my linguistic notion of the word "square" on the diagram,' we simply say 'I drew a square' and everyone knows what we mean.

Berkeley does hold, nonetheless, that the visual language of signs, like written or spoken language, is interpreted consistently, and that for the most part it is a reliable guide for understanding the natural world. Berkeley objects to Locke's version of ideas, and also to a Platonic conception of language (and perception) that presents signs as signifiers of transcendental objects or essential properties, but that does not mean he believes conventional signs are meaningless. The signs we interpret "are constant and universal," and "their connection with tangible ideas has been learned at our first entrance into the world; and ever since, almost every moment of our lives, it has been occurring to our thoughts, and fastening and striking deeper on our minds."[107] Heat may be a sign of fire, and fire itself is a complex visual sign, but although our words and ideas might never reveal the 'true natures' of the things they signify, Berkeley would not deny that 'heat' will always accompany 'fire' to every observer in every context, whether or not we choose to call the thing that burns us by some other name. The *relationship* between signs is not arbitrary, but rather learned, as Berkeley says, "by the slow steps of experience," which teaches us these relationships and around which we base the conventions of language. Indeed, Berkeley indicates that the visual language of signs encodes the rules we use to determine relationships, and provides the pattern on which we construct our spoken languages:

> Upon the whole, I think we may fairly conclude that the proper objects of vision constitute a universal language of the author of nature, whereby we are instructed how to regulate our actions in order to attain those things that are necessary to the preservation and well being of our bodies. . . . And the manner wherein they signify and mark unto us the objects which are at a distance is the same with that of languages and signs of human appointment; which do not suggest the things signified by any likeness or identity of nature, but only by a habitual connection that experience has made us to observe between them.[108]

Berkeley further develops this notion that vision is a language in his treatise *Theory of Vision, or Visual Language*, which despite being published nearly twenty five years after the *New Theory* reiterates most of the central points in the earlier work, and therefore indicates that Berkeley's attitude towards vision and language changed very little over the course of his life. The significant feature in both works is that Berkeley identifies vision as a 'language of nature' quite unlike the one Galileo proposed over a hundred years earlier. Berkeley's universal language teaches us how to read the connections between signs, but it does not reveal the universal architecture of the natural world. As he contends in *Theory of Vision*:

> A great number of arbitrary signs, various and apposite, do constitute a language. If such arbitrary connection be instituted by men, it is an artificial language; if by the Author of nature, it is a natural language. Infinitely various are the modifications of light and sound, whence they are each capable of supplying an endless variety of signs and, accordingly, have been each employed to form languages; the one by the arbitrary appointment of mankind, the other by that of God himself. A connection established by the Author of nature, in the ordinary course of things, may surely be called natural, as that made by men will be named artificial. And yet this does not hinder but the one may be as arbitrary as the other. And, in fact, there is no more likeness to exhibit or necessity to infer things tangible from the modifications of light, than there is in language to collect the meaning from the sound.[109]

What, in other words, does this 'language of nature' teach us about the true nature of objects, or the mind of God? Absolutely nothing, Berkeley answers, except insofar as it reveals that our minds have been created to interpret a world of linguistic patterns, conditioned by experience. We choose to associate the word 'green' with 'grass,' and the words we use are arbitrary and determined by convention. In a similar way, God has chosen to make whatever it is that produces the idea of 'grass' in our heads signify (among other things) what we interpret as 'greenness,' but this is a no less arbitrary connection, since the connection itself reveals nothing more than a relationship that is borne out by consistent experience. Berkeley's philosophy of knowledge is characterized by a persistent emphasis on relationships as the key to understanding, but Berkeley's understanding of these relationships is founded on a conventional linguistic structure, and not ontological necessity.[110]

Berkeley's Philosophy of Mathematics

Berkeley's linguistic and structural philosophy of representation and perception affected his approach to mathematics in a profound way. Indeed, his view of mathematics was a component of his general philosophy of representation, and his account both of the ontological status of mathematical objects and the epistemological legitimacy of mathematical demonstrations is entirely consonant with his approach to language. Furthermore, mathematics was a subject that received a systematic and thorough treatment in nearly all of Berkeley's philosophical works; as Jesseph notes, "his mathematical writings are not a loose collection of random thoughts on the subject," but rather his approach is "everywhere informed by a desire to avoid abstractions and to interpret mathematics as part of his general theory of signs."[111] This is a significant point, because we have seen that most philosophers of the period even in the nominalist tradition tend to treat mathematics as a special epistemological case. Not so with Berkeley, however, who attacked mathematics as vigorously as he did the conventions of vision and language, in "self conscious" opposition, again according to Jesseph, to the set of beliefs about ontology and epistemology that "was almost universally accepted by philosophers in the seventeenth and eighteenth centuries."[112]

Like most of the other philosophers we have considered, Berkeley distinguished between geometry and arithmetic (including algebra), although he considered both subject to the basic rules that formed the core of his anti abstractionist philosophy. Arithmetic, in Berkeley's view, is a purely formal language, since its object number "is entirely the creature of the mind," a position that reflects Barrow's philosophy of arithmetic.[113] Ideas of number, Berkeley contends, are not drawn directly from experience, but rather relate "to some particular combination of ideas arbitrarily put together by the mind," and are not signs of objects that have existence outside of the mind.[114] This is a position he first presents in the *New Theory*, where he compares the numbering of objects to the arbitrary names we give them, and maintains that such naming is done for 'convenience' rather than because of any necessary natural order. A 'single' idea, Berkeley points out (such as a 'house') is actually a composite of many related ideas, and what we call 'one' house is just a sign formed by the mind to represent the idea of that composite object.[115] To truly consider any idea as a single, concrete unit would require that we could abstract some essential quality that defines an object as a member of a general class—'houseness' from a collection of bricks, wood, and glass, in this case—which would be to hold an abstract idea. Because we can have no abstract ideas, and because therefore "there be no such thing as unity or unit in abstract," it follows that "there are no ideas of number in abstract denoted by the numerical names and figures," and numbers "can be supposed to have nothing as their object."[116]

To have 'nothing' as an object means simply that to think of a number is to think only of an arbitrary sign. Berkeley's approach to arithmetic is based on his thoroughgoing nominalism, in that he believes what we call numbers have no real ontological existence because they do not (and cannot) refer to particular concrete ideas drawn from experience. Berkeley does, however, grant that numbers (and therefore arithmetic) may be useful, despite being based on arbitrary signs: while they "are not regarded for their own sake," they do "direct us how to act with relation to things, and dispose rightly of them."[117] Arithmetic, in other words, is one of the logical ordering processes the mind uses to represent the relationships between ideas. Like language, arithmetic orders and associates our ideas, and it is valuable for communication and representation even if it bears no necessary relation to a real ontological order. In this sense, Berkeley is proposing that arithmetic is simply a formal language, and numbers are created by the mind to serve as marks or signs.[118] Berkeley's unapologetic nominalism, combined with a ruthlessly consistent application of this epistemology to all cognitive faculties, highlights what has been present in this tradition all along: an equation of mathematics with language and a denial of the ontological reality of mathematical objects.

Such is the case with arithmetic, at least; geometry, being putatively the science of visible extension, is slightly more complicated. While Berkeley argues that arithmetic symbols are signs devoid of any tangible signification, he argues that geometrical figures are representations of perceived objects in the world. What Berkeley challenges is the notion that the subject of geometry is abstract extension—which had been its traditional definition—for the obvious reason that such a conception violates his ban on abstract ideas. As he explains in the *New Theory*:

> It is commonly said that the object of geometry is abstract extension. But geometry contemplates figures: now, figure is the termination of magnitude; but we have shown that extension in abstract has no finite determinate magnitude; whence it clearly follows that it can have no figure, and consequently is not the object of geometry.[119]

Geometry, he is saying, is the science of *perceivable* extension, rather than of magnitude in the abstract; as a nominalist, Berkeley can accept that geometrical figures might be representations based on particular visual experiences, but not that they capture the general properties of collective groups of objects in nature (one cannot abstract the 'triangleness' from a particular three sided object). Names like 'triangle' may function as general signs that have other signs (but not real objects) as their referents, but this is purely a matter of convention. Similarly, we can call a three sided figure we draw a 'triangle,' but since finite extension exists only in our minds, such a representation has only an approximate correlation to signs formed from perceptions of other natural objects that we may, thanks to convention, determine are similar.[120] But we can only have ideas, according to Berkeley, of things that we have perceived, so all our ideas in geometry must be tied to particular experiences. In the case of standard geometrical figures, our ideas might simply come from the experiences we have had drawing on paper or reading a geometry textbook. Other classes of geometrical objects, however, like Descartes' chiliagon, are impossible, because as they are impossible to physically represent, they are literally unthinkable.

This serves as a general outline of Berkeley's linguistic and mathematical theory of representation, which was quite explicitly nominalistic. If Berkeley is notable for his willingness to take linguistic nominalism even farther than his predecessors, then he is equally noteworthy for his rigorous nominalist critique of contemporary mathematical practice. This is most evident in his strenuous opposition to analytic geometry and, in particular, the Newtonian calculus, against which he devoted several incisive and polemical treatises. However, before considering this critique which offers the clearest indication of the impact of nominalism on the development of modern mathematics we will examine the mathematical philosophies of several important English mathematicians. This group includes Barrow and Newton, whose approaches though not precisely nominalistic bear the distinctive influence of the seventeenth century nominalist/constructivist tradition.

4 Three Mathematicians: Constructivist Epistemology and the New Mathematical Methods

In considering the relationship between nominalism and the epistemology of representation in the seventeenth century, a central theme that has returned again and again is 'constructivism': the idea that general knowledge is constructed by the human mind out of discrete individual experiences, and is therefore to some degree 'artificial.' For Gassendi, Hobbes, and Berkeley, constructivism offers a way of explaining the formation of general concepts without relying on Aristotelian universals. For the nominalists, constructivism also provides an epistemological framework for discussing mathematics that allows the existence of generalized classes of objects (such as geometrical objects) without necessitating that those entities be considered ontologically real. Nominalism and constructivism have thus been intertwined in an epistemology that opposed a very common contemporary belief (expressed, for example, by Descartes) in the strict ontological reality of both mathematical objects and universal categories. The inevitable skepticism attached to nominalist/constructivist epistemology also challenged the notion that nature is a divine architecture of archetypal forms, universal objects, and mathematical entities that are directly apprehensible by the human intellect.

This chapter will survey the epistemological orientations of three important English mathematicians John Wallis, Isaac Barrow, and Isaac Newton who occupy distinct positions on a spectrum of belief between realism and nominalism. Although none of the three can be called an outright nominalist (though Barrow comes closest), all three exhibited tendencies towards a constructivist philosophy of mathematics, though each manifested this tendency quite differently. Wallis, who was the most sympathetic towards analysis in the group, had comparatively little interest in the ontological foundations of mathematics. Barrow, on the other hand, devoted much energy to considering the epistemological and ontological basis of mathematics, and developed a position of epistemological constructivism reminiscent of Gassendi's. Finally, Newton stands as a problematic and fascinating character, torn between Barrowian constructivism and ontological realism, and between traditional geometry and analysis. Ultimately, Newton's reconception of the grounds for mathematical natural philosophy, and of the entities that populate the natural world (forces operating in a framework of absolute space) transcended the categories established by realists and nominalists of the preceding decades of the seventeenth century. I will offer no definitive characterizations of any of the three mathematicians in this chapter; rather, I will cite them as examples of a growing tension in early modern mathematics between realism and nominalism, and I will examine the role of constructivism as a philosophical *media via* between these two opposing poles.

Mathematization and Constructivism

The traditional paradigm established by E. A. Burtt and Alexandre Koyré, which saw a progressive rise in a homogeneous tradition of 'mathematization' through Galileo, Descartes, and Newton, is no longer sufficient to explain the development of modern mathematical philosophy. As Michel Blay has argued, it is more accurate to describe two distinct ontological approaches to mathematical philosophy in this period, one based on a realist geometrical ontology, which he terms 'geometrization,' and the other based on analytical techniques, which he terms 'mathematization.' According to Blay, the distinguishing feature of mathematization is that mathematical entities themselves are not considered ontologically real; rather,

> [its] object is to reconstruct the phenomena of nature within the domain of mathematical intelligibility in such a way that these phenomena find themselves governed by quantitative laws which can be exploited for the purpose of predicting the course of nature by means of mathematical reason.[1]

Such reconstruction is consistent with the definition of mathematical constructivism I presented earlier, which distinguishes between the mathematical construction of nature and some real ontological order. In the cases of two important mathematical practitioners of the later seventeenth century Barrow and Newton this approach to mathematical constructivism is linked with a 'physicalist' conception of geometrical magnitudes, and also, by extension, to important elements of nominalist ontology.

There are three distinct meanings of construction that can be applied to mathematical philosophy: On one level, construction can refer to the generation of curves by motion, a technique employed by mathematicians from Archimedes through Newton. This technique has ontological significance when used by Hobbes and Barrow the mechanical aspect of Hobbes's geometry, for example, is an important component of his general materialism but strict mechanical constructivism does not necessarily imply broader epistemological or ontological consequences. A second, more general definition of mathematical constructivism holds that to assert "that there exists a mathematical object (such as a number) with a given property is an assertion that one knows how to find, or construct, such an object."[2] This definition does not limit 'construction' to the mechanical generation of curves, but rather posits that all mathematical objects have properties determined by the mathematician, and opposes the realist position that mathematical objects have a mind independent existence.[3] The second definition fits more closely with the sense of Blay's statement quoted above, which implies that the mathematical construction of nature is distinct from some real ontological order, a step Blay sees as crucial in the transition from geometrization to mathematization.

Finally, constructivism can be understood in an even broader sense, to mean a philosophy of human understanding that is based on the mind's ability to construct general knowledge from individual experiences. This 'epistemological constructivism' is characteristic of the philosophies of Gassendi, Hobbes, Locke, and Berkeley, and, I will argue, Barrow, and for the purposes of this book is the most significant meaning of the term. There are important connections, however, between broad mathematical constructivism and epistemological constructivism, and nominalist philosophy of mathematics tends to conflate the two, producing a position that mathematical statements have validity because they are drawn from logically consistent systems of axioms and definitions whose rules were drawn up by human beings. At the center of this issue is concern about the ontological status of mathematical objects, or in other words the justification for mathematizing the natural world.

Peter Dear, in particular, has studied the history of a tradition in mathematical constructivism in seventeenth century natural philosophy, and his analysis supports my broad contention. Dear connects the later seventeenth century constructivist tradition (in which he includes Barrow and Newton) with the work of late sixteenth century mathematical practitioners such as Christoph Clavius, and he concludes that constructivism influenced a significant agenda in mathematical natural philosophy during the Scientific Revolution. As Dear explains:

> Newton's work drew directly from the tradition of mathematical sciences {of Christoph Clavius and the Coimbra Jesuits} . . . and it is shown further to be premised on a constructivist conception of mathematical objects themselves. Geometrical figures, according to a dominant line of argument in the seventeenth century, were things *to be drawn* rather than pre existing in a Platonic realm.[4]

Here Dear connects strict mechanical constructivism with more general mathematical constructivism: the fact that objects (like curves) are "to be drawn" implies that they do not have mind independent existence. This allows a comparison although one not explicitly made by Dear between the tradition of mathematical constructivism Dear describes and the nominalist tradition of epistemological constructivism. It is a major goal of this book to connect these two distinct constructivist epistemologies, and I argue that nominalism is a vehicle for understanding how mathematical constructivism became part of a wider philosophical debate in the seventeenth century.

By the mid seventeenth century, mathematical practitioners had become increasingly committed to the use of analytic geometry, infinitesimal techniques, and algebraic notation, all of which added greatly to the power of mathematical natural philosophy, but which also presented a number of epistemological and ontological questions.[5] These questions concerning the nature and reality of mathematical objects, the truth of mathematical demonstrations, and the proper 'language' for mathematical discourse to a great extent overlap with the issues central to the more general debate between realists and nominalists. Mathematics became an important battleground on which this conflict took place. Mathematical practitioners developed a number of strategies for justifying the use of increasingly abstracted and formalized analytic techniques, but as a number of historians have shown, the transition from a mathematics oriented towards traditional Euclidean geometry to one centered on analysis was by no means uncontroversial.[6] The tension between Hobbes and Wallis, for example, was felt (albeit often less dramatically) by many others in the wider mathematical community, and particularly in Britain discussions of the 'usefulness' and legitimacy of analysis continued into the eighteenth century.[7]

It should be noted, however, that there was considerable precedent for this discussion, not all of which relates directly to the central themes of nominalism this book is concerned with. For example, Jacob Klein has traced the development of mathematical philosophy from antiquity through the seventeenth century, using François Vieta as the primary early modern conduit for a new philosophy of number. Vieta, who during the late sixteenth century developed one of the most influential methods of symbolic algebra, helped pave the way for a formal interpretation of mathematics by introducing a new ontological interpretation of the numerical 'species.' As Klein maintains:

> the being of the species in Vieta, i.e., the "being" of the objects of "general analytic," is to be understood neither as independent in the Pythagorean and Platonic sense nor as attained "by abstraction" . . . i.e., as "reduced" in the Aristotelian sense, but as *symbolic. The species are in themselves symbolic formations namely formations whose merely potential objectivity is understood as an actual objectivity.*[8]

Klein argues that this conception ultimately led to a modern conception of number, since numerical species became "comprehensible only within the language of *symbolic formalism* . . . As soon as 'general number' is conceived and represented in the medium of species as an 'object' in itself, that is, symbolically, the modern concept of 'number' is born."[9] It may indeed be the case that the symbolic ontological interpretation of number owes as much (or more) to the tradition Klein describes as it does to the influence of nominalism; nonetheless, I will argue that the nominalist critique of realist ontology should also be considered an important factor when examining discussions of mathematical ontology among later seventeenth century mathematicians.

The views of the three mathematicians to be treated in this chapter may be summarized as follows: Wallis, on the whole, interpreted analysis as a heuristic device, but his statements about the nature of truth and certainty make it clear that he did not believe mathematics provided *a priori* deductive certainty about the nature of reality. Furthermore, in his writings on experimental natural philosophy Wallis held firmly to an inductive and empirical approach, and there is evidence that this methodology informed his epistemological views about mathematics. Barrow, on the other hand, was a die hard supporter of the synthetical geometrical approach, and argued at length against the validity of analytical methods. These arguments were, however, targeted at the ontological basis for analytic geometry, and his objection to the reality of algebraic demonstrations is wholly consistent with the nominalist epistemology I have described. Barrow's philosophy of geometry incorporated elements of mechanical, mathematical, and epistemological constructivism, and his basic epistemological stance was quite similar to Gassendi's. Finally, in Newton, we are presented with a problematic figure: although as a young man he was exposed to and influenced by nominalist philosophies (Gassendi, via Walter Charleton, was an important early source), his ultimate ontology of space and force fits neither realism nor nominalism. Rather, Newton's conception of force is a new kind of philosophical entity, neither traditional geometrical 'object' nor artificially constructed concept. Though Berkeley's treatment of the calculus gave Newtonian mathematics a nominalistic critique, Newton's mathematized physics points to a breakdown in these medieval and early modern categories of explanation.

John Wallis

John Wallis was born in 1616 and lived a long and extremely productive life, eventually publishing over a dozen significant mathematical works and authoring more than fifty papers in the *Philosophical Transactions* of the Royal Society, which he helped found. By the time he died in 1703, Wallis had lived through several generations of mathematical and philosophical practice. He was an avid supporter of the new analytical and infinitesimal methods, and both his *Mathesis Universalis*, a treatise on the universal applicability of symbolic or 'specious' algebra, and his *Arithmetica Infinitorum*, a work on the properties of infinite series, are regarded as forerunners to the calculus.[10] Because of his long life and career, Wallis can also be regarded as an indicator of mathematical opinion at the height of the Scientific Revolution, since he was exposed to the full spectrum of mathematical and philosophical beliefs described in this study.

While Wallis is best remembered for his mathematical treatises, he was quite a prolific author on other subjects as well, and in addition to penning notes on observations and experiments, he also produced a number of theological and metaphysical works.[11] Wallis was also notorious throughout his career for his acid pen and willingness to engage in controversy (as is evidenced by the Hobbes affair), and the earliest example of this tendency was his response to a 1641 treatise by Robert Brooke entitled *The Nature of Truth, its Union and Unity with the Soule*. In that work, Brooke proposed that the essence of truth is drawn by a kind of Platonic light emanating from the 'soule' of God, and that human reason, which flows from this same source, joins human souls with God.[12] For reasons that are not entirely clear, Wallis was sufficiently annoyed by this work that he published a response, which he called *Truth Tried: or, Animadversions on a Treatise Published by the Right Honourable Robert Lord Brook, Entitled, The Nature of Truth, Its Union and Unity with the Soule*.[13]

Truth Tried, though not explicitly concerned with mathematics, reveals some of Wallis' deeply held epistemological beliefs with a clarity and candor not found in his later mathematical works. It might be argued that, given the early date of this work, the ideas it expresses cannot be relied upon as indications of Wallis' mature thought, but I will argue otherwise. Wallis had, by his own admission, already become grounded in logic, physics, and metaphysics, and had as well "imbib'd the Principles of what they call the *New Philosophy*" while a student at Cambridge.[14] Wallis built upon the foundation of these interests throughout the rest of his life, but in the case of mathematics he was a relative latecomer: not until 1649, when he took a position at Oxford to teach geometry, did mathematics "which had before been a pleasing Diversion . . . [now become] my serious Study." Wallis was motivated to explore the foundations of mathematics in the same manner that he practiced all natural philosophy, which was "to examine things to the bottom; and reduce effects to their first principles and original causes."[15] Wallis' mathematical studies were, in other words, an outgrowth of his general interest in natural philosophy, and to this end he remained true to the principles of empirical investigation and inductive reasoning in which he had become grounded as a young scholar. *Truth Tried*, published in 1642, is indeed a youthful work, but it is the youthful expression of a philosophy that changed remarkably little in its principal aspects over the great length of Wallis' career.

The epistemology of *Truth Tried* resides somewhere between Gassendi's and Hobbes's radical skepticism and the rational certainty of the Cartesians. Several times Wallis identifies Brooke's philosophy with Platonism, and he makes it quite clear that he opposes the belief that truth is resolvable to abstract, archetypal images floating outside the world of sense perception.[16]

According to Brooke, truth is an emanation from the divine source that the mind inherently perceives; true statements are true because their objects are universal archetypes, analogous to Plato's forms. Wallis dismisses this notion of truth as uselessly speculative: "The doctrine of the *Platonists*, reducing all *Being* to *Number*, must either be taken in a Metaphysical, Analogicall sense, or not taken at all."[17] According to Wallis, the most useful logical demonstrations are statements of 'logical' or 'physical' truth, which draw veracity not necessarily from agreement with the essential nature of an object, but rather from agreement with terms in a definition. Beyond 'metaphysical' truths:

> There is yet another *Truth*, and you may call it a *Physicall* Truth, a *Formall* or *Essential* Truth: Thus that which hath the Essentials of a Man, is *vere Homo*; so an Infant is a *true* Man. . . . thus a *Syllogisme* in a right form, is a *true Syllogisme*, though the Proportions be false. And the like.[18]

The 'essence' of an object, in this case, is not some universal archetype, but rather the formal characteristics that define the object logically as such. This is because what Wallis like Gassendi, Hobbes, and Locke believes to be the 'object' of a logical demonstration is not the actual physical object being considered *per se*, but rather a representation of that object formed in the mind. Nor, Wallis maintains, are ideas found in the mind innate or implanted at birth:

> Yet I cannot with his Lordship subscribe to the *Platonists*, to make *Knowledge* nothing but a *Remembrance*. (As if there were naturally in our Understanding, the Pictures or Pourtraictures of all Truths) . . . I approve rather of *Aristotle's Rasa Tabula* . . . making the Understanding, of itself, to have no such *Idea* or Picture at all, but capable of all.[19]

Ideas originate, then, as the products of sense experience, where the mind forms representative copies from objects it perceives. It is these representations that are the currency of thought. If the representations bear no relation to the physical objects as experienced, then they are 'false copies,' but they nonetheless have a kind of existence. As Wallis explains:

> when the Intellect doth Understand, it frames an *Idea*, a picture, or representation of the Thing understood; which Picture, or *Idea*, is a *Reall* Picture, (it hath the Truth of Being) whether it have the Truth of Representation or not; that is, whether it be Like or Unlike, whether it Agree or Disagree, with the copy or object which it represents. When the Understanding conceives an *Ens Rationis*, the *Idea* or *Conceptus* is not this Imaginary Being . . . But the *supposed Object* of this Conception . . . When a Painter describes in a Table some Antick Shapes or strange

> *Chimera's*; his Description, his Draught, is not a Fiction, but as Reall as the true Pourctraiture of a living Man: but that by which this description is represented, *that* is the Fiction.[20]

The 'truth' of an object's 'being' lies in how well the representation satisfies the conditions of its definition: ideas of 'horse' and 'mermaid' have equal 'essential truth,' since the essence of a representation is ultimately its definition. This is particularly the case with what Wallis calls 'logical truth,' or "the Truth of a *Proposition*," which "hath nothing to do with the *Reality* either of the *Object* or of the *Act*." This designation concerns statements we make in logic and language, where "a True *Proposition* may be framed concerning an Imaginary *Object* . . . And an *Act* Metaphysically True (a Reall act) may be Logically False," since "Logicall *truth* and *Falsehood* . . . have not an Absolute Being, but Relative."[21] In other words, Wallis is suggesting that ideas which exist in the mind may be 'true' or 'real' without necessarily corresponding directly to ontologically real extra mental objects. This position is surprisingly similar to Gassendi's view, and positions Wallis within the broad tradition of epistemological constructivism common to Hobbes and Locke as well.

Though an early work, *Truth Tried* helps to put Wallis' later, more explicitly mathematical works, in context. In the sense that he favored the 'new' methods in mathematics, Wallis has been considered a 'modern' mathematical thinker: as Pycior claims, "Wallis was responsible for assuring that by the late seventeenth century English algebra was 'early modern' in every respect, not only in its language but in its generality, universe of objects, and methods as well."[22] Wallis has also been identified as a modern for his association with the Puritan revolutionaries who populated the early Royal Society, as is argued in R. F. Jones's classic, *Ancients and Moderns*.[23] But as Katherine Hill has shown, Wallis did not consider algebra to be a 'new' subject in opposition to the 'old' discipline of geometry; rather, "for Wallis, to prefer algebra was simply to favour one branch of classical mathematics over another, not to make a radical break with tradition."[24]

Wallis' preference for the symbolic form of analytic geometry does set him apart from Hobbes and Barrow in a significant way, although I will suggest that this distinction does not cut as deeply on ontological and epistemological levels as one might expect. In terms of Wallis' views on these philosophical issues, a starting point is again the early *Truth Tried*. His treatment of mathematics in that work is consistent with a constructivist philosophy of truth and representation, and his description of the nature of number is particularly interesting. Here Wallis proclaims that number, like truth and falsehood, is based in relation:

> Neither will I stand to debate the controversie, concerning the Nature of *Number*, whether it be a *Reall* or *Rationall* Being; Which I conceive to

> have as much Reality as a *Relation* hath, and no more; that is, hath *Fundamentum in re*, but in its *Formality*, it is onely *filius noentis*, the birth of Reason.[25]

This somewhat cryptic statement deserves a close reading. In the first place, in declaring that he is not interested in joining the debate over the ontological status of number, Wallis gives a clue about where he stands on the relation between the philosophy and the practice of mathematics. He has a position, but regards the arguments surrounding it (and perhaps he is thinking specifically of Gassendi, whom he has read, and Hobbes) as tiresome and distracting. Wallis, being more inclined towards practice, is expressing some of the frustration commonly found when practicing scientists confront the philosophy of their discipline: he can see the problem, but isn't sure he sees the point. Wallis is perhaps also being slightly disingenuous: he spends a hundred pages debating the abstract nature of truth in *Truth Tried*, so obviously he isn't averse to philosophical introspection. Quite likely by dismissing lengthy philosophical arguments on this subject Wallis is hoping to brush aside a deeper critique of the ideas he himself is presenting: by making the issue seem simple and resolved, he is asking his audience to take it on faith that it is resolved.

In the next clause, however, in which Wallis calls number a relation, a greater ambiguity arises. The very grammar of this passage is problematic: is Wallis saying that it is "the controversie" or "the Nature of *Number*" that has no more reality than "a *Relation*"? The first interpretation, while grammatically plausible, is incoherent. Wallis may believe that the arguments provide little to recommend one position over the other, but to make this point in such an oblique way (by referring indirectly to his theory of logical truth and falsehood, which are formal distinctions) seems ludicrous. Furthermore, he makes this point about number some fifty pages before he discusses truth as a relation. That leaves the second interpretation, that it is the nature of number itself that is a relation and it is significant that Wallis writes that number has "as much" reality as a relation, and also "no more." As to what precisely he means by this, Wallis goes on to distinguish the 'being' of the nature of number from its 'formality.' Like other objects perceived by the mind, number has 'foundation in the thing,' or 'metaphysical being,' but as it is perceived in its formal quality, it is "the birth of reason," or in other words 'born of the mind.' Like other natural objects, we perceive numbers as representations which are artificial that we construct by passing sense perceptions through cognitive faculties, and the result is a formal, and not an essential knowledge of that object. This is why, when describing the nature of mathematical phenomena in nature, Wallis excludes the application of mathematical qualities from the essences of phenomena: "As for the *Mathematicall* Unity of Harmony, Proportion &c. It being only Relative (for they denote but the Relation of one thing to another) can conferre nothing to the Constitution of an Absolute Essence, as of Sounds &c."[26]

The conception of number as relation is not unique to Wallis similar ideas are to be found in Barrow's and Newton's work. It answers the question of whether numbers are ontologically discrete entities or mental fictions by hedging somewhere in between: numbers (i.e. units) are not necessarily discrete entities, but rather are symbols that stand for proportions or magnitudes that do in some sense have reality by virtue of their being constructions drawn from physical experience. This idea is reflected in Newton's fluxional calculus, and it is not surprising to find a version in Wallis' philosophy, given his role in developing the foundations of that mode of analysis. What is interesting is Wallis' suggestion that these entities have a merely formal component. The issue at hand, of course, is whether formal representations in mathematics have ontological existence; to this question the nominalists have consistently said 'no.' As can be inferred from *Truth Tried*, Wallis is hesitant to make ontological claims about purely rational entities: things that have only 'logical truth' do not as such have 'truth of being.' But Wallis also acknowledges that actual natural objects do have being 'as such,' though our understanding of their real existence is limited to representations of them as formal entities. Are mathematical objects like other natural phenomena, or are they rather like words, which are arbitrary signs operating in a formal system?

This is not a question that Wallis answers with either a definitive yes or no. In general, Wallis' philosophy of mathematics corresponds with his philosophy of logic and language: on a purely formal level, statements made in symbolic analysis are justified by their rules and axioms, and the truth that is produced is 'logical' rather than 'metaphysical.' But on a more general level, Wallis clearly believes that mathematical knowledge can be made to correspond with knowledge of the actual physical world, which is what makes mathematics useful in natural philosophy. In *Mathesis Universalis*, Wallis begins Chapter 3 under the title "Of Mathematical Demonstrations: Whence from Mathematics true Science is Determined," and goes on to contend that "when rightly constructed," mathematical demonstrations can reveal knowledge which otherwise "no one can perceive." He does however qualify this statement by declaring that "in mathematical bodies . . . not everything exists in the natural world," such as the exact lines and surfaces of geometrical objects.[27]

It is important to separate Wallis' approach to algebra from his view of geometry. In his mind, these two branches treat the same basic subject, but whereas geometry "at most {explains} the truth of the Proposition as to one case" and is useful when "such Lines and Figures be necessary where the Truth of a Proposition depends on Local Position," algebra gives statements that may be applied to all cases, and not just a particular problem at hand.[28] Epistemologically, however, Wallis acknowledges that algebra and geometry are quite different: he defines algebra as analysis, "Which imports a Resolution of dissolving, of what is supposed to be compounded or made up," which is opposed to the synthetic art of geometrical construction.[29] It is for this reason that Wallis claims that analysis allows for the discovery of new knowledge: in answering Pierre de Fermat's concerns about the method of indivisibles, Wallis replies, "he doth wholly misstate the design of that Treatise [*Arithmetica Infinitorum*]; which was not so much to show a Method of Demonstrating things already known . . . as to shew a way of *Investigation* as finding out things yet unknown."[30] But given that algebra is universally generalizable and that it produces new knowledge, what is the status of the results produced?

In the first place, Wallis defends algebra on merely practical grounds, since its characters can be substituted for known and unknown magnitudes otherwise represented by specific geometrical figures or numbers. This is a mnemonic aid, a way "to make a choise of such notes or species as may some way represent to the Memory or Fancy the Quantities designed by them." Wallis, however, rejects any notion that the specific characters used have any significance in and of themselves: "such choise of Notes do not at all influence the Demonstration, yet doth it assist the Fancy and Memory, which would otherwise be in danger of being confounded in a Multitude of Symbols."[31] This means, however, that Wallis will make no ontological claims for the visually represented elements of his mathematical analysis: unlike a 'circle' or a 'tetrahedron,' an algebraic character has no claims to being a direct representation of the object it purports to signify. Algebra is 'semiotic' in the sense that its characters are signs that have meaning only because people have chosen to attach particular reference to them. Algebraic statements are meaningful enunciations only so far as these signs are interpreted and the grammatical rules of their combination are understood by the reader. In this sense, the parallel between algebra and structural linguistics (both of which Wallis favored) is clear.

The built in arbitrariness of algebra also allows for the expression of statements that are not necessarily reflective of the 'real world'; just as we can make statements in language that are logically consistent but 'metaphysically' false, impossible and irrational numbers can be represented in algebra. For epistemological justification, we should recall Wallis' explanation in *Truth Tried* that "*Logicall Truth* . . . hath nothing to do with the *Reality* either of the *Object* or the *Act*."[32] To say something is 'true' in this context is not the same thing as saying it is 'real.' At the same time, however, Wallis is moved to justify his algebraic demonstrations in physical terms. He even extends this to 'impossible objects' like negative numbers:

> Yet is not that Supposition (of Negative Squares,) either Unuseful or Absurd; when rightly understood. And though, as to the base Algebrain Notation, it impute a Quantity less than nothing: Yet, when it comes to a Physical Application, it denotes as Real a Quantity as if the Sign were it; but to be interpreted in a contrary sense.[33]

Using the example of a negative number, Wallis draws a physical picture where 3, for instance, describes the motion of a man who, walking forward then backwards, ends up three units back from the place where he started. This kind of spatial conception of negatives has become universally familiar following the invention of the Cartesian coordinate system. But this case "when rightly understood" also mitigates any ontological basis for interpreting the quantities represented by mathematical symbols as 'real' objects: here 3 is not an independently real entity, but rather a "mean proportional" between two arbitrarily selected points on a graph. The statement '5 8 = 3' might therefore refer to some ontologically real measurement in nature, but it is at its heart a narrative description of an event, and not the definition of a physical object.[34]

Wallis is ultimately concerned with having his mathematics refer to the physical world, but he does not necessarily require that it captures its essence in a metaphysical sense. Wallis notes that analysis is ideally suited to abstraction, because the very language of algebra frees the mathematician to consider only the arithmetical qualities of "Proportions, and regular Progressions" without troubling "to affect Lines and Figures" as the geometricians do.[35] Since Wallis conceives number as a 'relation' and he is primarily interested in describing general rules about the proportions between magnitudes, synthetic geometry, being an attempt at visual representation, is not as free "from the impertinences of the matter" of the objects it describes as is algebra. Geometry, in other words, is abstract, but not abstract *enough*: the lines and figures of geometricians are caught between trying to represent nature in its particularity and giving an abstract general representation. Indeed, Wallis calls "such Arithmetical Demonstrations" on which algebra is based "more Abstract, and therefore more universally applicable to particular occasions" than geometrical ones, and credits this abstractness for the success of his method of treating infinite series.[36]

At the same time, however, Wallis does not claim that abstraction reveals the essence of nature, as Descartes and others argued. Wallis' conception of abstraction is again similar to Locke's: abstraction is a logical or rational process by which the mind separates certain qualities from perceptions of objects and treats those qualities in a formal sense. "Mathematical Definitions," Wallis notes, "are not to be thus restrained to the Physical essence of the things Defined," since such a quality as "Metaphysical Essence," even if apprehensible, "in itself may be disputable, and not so easily demonstrated when there is occasion."[37] This gets at the heart of Wallis' attitude towards the application of mathematics to the physical world: we use our senses to perceive the qualities of particular objects, then process this data in the mind to form ideas, from which abstract representations may be constructed that are described in the language of mathematical demonstration. This is undeniably a process of translation, as the final product in no way claims to be an exact description of a natural phenomenon; algebra itself is a language of translation, since physical characteristics and (in the case of analytic geometry) geometrical representations are converted to the logical system of arithmetic, and expressed in the symbolic nomenclature of analysis. Algebra is not a ‘construct’ in the sense that geometry is (i.e. a synthetic construction of particular figures from general rules and axioms) but in another sense is an ‘artificial’ mode of expression, in which human reason constructs a representation of reality that does not necessarily exist in nature.

Although Wallis is not a nominalist, his mathematical epistemology and philosophy of representation, I argue, are broadly constructivist. Looking forward to Barrow and Newton, another point of similarity is Wallis’ continued reference to his method of mathematical demonstration as “inductive.” Wallis claims that his arithmetic of infinites is “demonstrated by way of Induction,” which method “shews the true natural investigation” that is, it builds a case on observations of particular examples that demonstrate a general rule about a particular class of phenomenon.[38] In practice, this means that it is not enough to base a mathematical understanding of physics on a single definition or axiom; rather, definitions are built from a body of ‘observations’ (or individual examples), which provide evidence of the generality of a rule. Wallis seems confident that “after such Induction continues for some few steps” one is justified “to conclude universally,” but this confidence is necessary for practical applications, since “without this, we must be content to rest at particulars (in all such kind of Process), without proceeding to the Generals.”[39]

This approach, as we will see, figures prominently in Barrow’s philosophy of mathematics, and mitigates some of the distinctions historians have, in the past, drawn between Wallis and Barrow. Hill observes that between Wallis and Barrow (whom I consider next) there is “a tension between the emerging algebraic techniques and the more traditional geometric mode of thought.”[40] Wallis’ mathematics certainly incorporates epistemological concerns that are modern in flavor, but, as I will argue, Barrow also expressed these concerns in his geometrical philosophy of mathematics. The mode of mathematical expression adopted by philosophers of this period did not necessarily determine the epistemological content of their work; there were rationalists who favored analysis (Descartes, Leibniz) and also traditional geometry (Antoine Arnauld), as well as constructivists like Wallis and Barrow who used opposite methods. Wallis, with his emphasis on analysis, was disposed towards formalism, which allowed for abstract representations and general categories so long as it was recognized that these representations had limited ontological content. Barrow, as we will see, regarded analysis with suspicion and was unable to trust a formal system because of his fear of misrepresenting the ontological basis of the objects he was describing. Both men, however, attempted to present philosophies of mathematics that were compatible with the new experimental natural philosophy.

On the broader issue of the mathematization of nature, Wallis’s philosophy supports Blay’s claim that “the mathematization of natural phenomena, while retaining the initial insistence on the requirements of deductive organization inherited from the Euclidean geometric model, put aside any meaningful ontological claims it might have had."[41] Mathematical demonstrations, in other words, were seen less and less to be ontologically real statements, and were treated increasingly as formal, constructed, and even narrative descriptions of nature. This claim is at least partially borne out by Wallis' example. In the language of natural philosophy, we increasingly see terms such as "measurement" applied to mathematical statements, as in a letter to Oldenburg of 1668 where Wallis claims "what is Physically performed, is Mathematically measured . . . there is no other way to determine ye Physical Laws of Motion exactly, but by applying ye Mathematical measure and proportions to them."[42] This corresponds with Wallis' and others' conception of number as a relation or proportion between magnitudes, of mathematical demonstration as inductive, and of mathematical statements being descriptions of physical events (which is also reflected in Hobbes's, Barrow's and Newton's idea of geometry as a mechanical art). To Wallis, mathematical statements were descriptions of reality constructed to meet formal and logical requirements; despite their often expressed (and mutual) dislike, Wallis and Hobbes had more in common then either was ever likely to admit.

Isaac Barrow

Most of what we know about Isaac Barrow's life and beliefs comes by virtue of his interaction with Newton, while the latter was a young scholar at Cambridge. It had traditionally been assumed that Barrow was Newton's tutor, but recent scholarship has determined that this was not in fact the case. Barrow certainly did befriend Newton and he appears to have provided one of the only stable relationships in Newton's early life, but the relationship was more that of senior to junior scholar than teacher to pupil.[43] J. M. Child, who in 1916 published a translation of Barrow's *Geometrical Lectures*, made one of the earliest historical assessments of Barrow's career. In the Preface, Child made the bold claim that "Isaac Barrow was the first inventor of the Infinitesimal Calculus," which Child based on his own conversion of Barrow's geometrical proofs in the *Lectiones Geometricae* into algebraic form. Despite the admitted anachronism of expressing Barrow's ideas in a form and notation he expressly rejected, Child felt justified in stating that "Barrow was writing a calculus and *knew that he was inventing a great thing*."[44] Although this view has enjoyed some popularity over the years, as Michael Mahoney notes, a more nuanced understanding of Barrow's and Newton's mathematical development has led historians to conclude rather that Barrow was not interested in "anything smacking of infinitesimals or limits," and that Barrow's research "does not encompass the program that led Newton and Leibniz to the calculus."[45]

Nonetheless, Barrow was an important and respected mathematician in his day, and his work is significant in the history of early modern mathematics.

I will not analyze Barrow's technical work in detail, but rather I will focus on the epistemological and ontological claims for mathematics made in his philosophical writings. Barrow's writings, most of which were first delivered as undergraduate lectures, are general in scope, philosophical in character, and directly explore the foundational questions in mathematics with which this book is concerned. There are a variety of reasons for this, but as Mordechai Feingold points out, one reason Barrow was so concerned with justifying the philosophical status of mathematics was that as the first Lucasian Professor at Cambridge he was charged with 'selling' mathematics to undergraduates who might otherwise choose another field.[46]

During his short tenure as Lucasian Professor, Barrow delivered three sets of lectures in 1664, 1665, and 1666 which were subsequently published in partial form in 1683 as *Lectiones mathematicae* and as a complete set, translated into English, in 1734. There are also two other sets of lectures, the *Lectiones opticae* of 1669 and *Lectiones geometricae*, published in 1670; together, these writings comprise Barrow's entire mathematical *œuvre*.[47] It is important to recognize that the mathematical lectures were designed as an introduction to mathematical concepts for undergraduates who had little formal training in mathematics, and as such, the lectures do not delve deeply into technical issues (which the *Lectiones opticae* and *Lectiones geometricae* do to a much greater extent). Barrow seems to have had two mutually reinforcing pedagogical aims in his Lucasian lectures: first, to tempt students who otherwise might turn to medicine or law to consider the study of mathematics (which Barrow worried was under subscribed), and, second, to justify the importance of mathematics as a legitimate branch of philosophy. In his "Prefatory Oration," Barrow notes the advantages of mathematical training for the development of the mind, citing the "Spur to our Reason which accrues from this Mathematical Exercise . . . to dispute strenuously as well as judge solidly with a Readiness of Invention, a Justness of Method, and Clearness of Expression."[48] Here we see Barrow's interest in approaching mathematics as a branch of logic; as Pycior contends, Barrow was instrumental is laying "the groundwork for the defense of mathematics as a logic," to which end he "extolled geometry as the 'perfect logic.'"[49]

Indeed, Barrow presents his discussion of the foundations of mathematics alongside an explanation of the foundations of reason, language, and representation. Like most commentators of his era, Barrow claims that mathematics "depends upon Principles clear to the mind" and "draws certain Conclusions," but like his nominalist contemporaries, he also rejects realism and Platonism as the foundation of that certainty.[50] Rather, he proposes a kind of mitigated conventionalism that makes him less extreme than Gassendi or Hobbes, but somewhat more nominalistic than Wallis.

Barrow's philosophy of representation is developed mostly through his philosophy of mathematics, but on a few occasions he makes general remarks about the nature of the understanding, as in this discussion of the relationship between names and the objects they describe:

> Hence we perceive that the Truth of a Definition is merely arbitrary as to its Subject. For that is no more than a mere Name imposed contingently, and at pleasure; being either feigned by the Teacher of the Science of his own Authority, or taken from common Use, or approved by the Consent of the Learned; and no where signifies the Thing defined by its own Nature, but altogether from the Institution of Men . . . the Truth of any Definition depends upon the Possibility of the Hypotheses, which it includes, so that to make a Definition true, the Name assigned must belong to a Thing following some possible Supposition expressed in it.[51]

The starting point for Barrow's epistemology is the assumption that "the mind may be sufficiently instructed without them [innate notions], and by its Native Power acquire to itself the necessary Principles and Means of Science," which hints at his commitment to empiricism, as well as to a certain kinship with Wallis, who also saw the mind as a "*rasa tabula*."[52] Late in the first set of mathematical lectures, Barrow describes the process by which knowledge is acquired, which he claims is "the Original of all Natural Science, and genuine Method of reasoning even from the first Foundations of Knowledge":

> The Mind, from the Observation of the Things objected, takes Occasion of framing Ideas, which, as soon as it clearly perceives to agree *with the things that may exist*, it affirms and supposes; then appropriating Words to them forms Definitions, and from the Consideration and Comparison of these together it draws Consequences and makes Theorems, which being joined together into certain Systems do compose particular Sciences.[53]

Barrow clearly recognizes a conventional aspect in the naming of objects of knowledge, but he also expresses the positive notion that ideas can agree with things in the world. Note, however, the qualifiers in this and the earlier quotation: a name "must belong to a Thing following some *possible* supposition," and "ideas agree with the things that *may* exist." The significance of this point will shortly become clear.

These issues relate to Barrow's philosophy of mathematics in a crucial way: in contrast to Wallis and many other leading mathematicians of his day, Barrow favored traditional Euclidean geometry over analysis, a position he justified on the basis of geometry's supposed superiority in representing the physical aspects of nature. Like Hobbes, Barrow conceived of geometry as a "sense based" science that was most proper for representing empirically perceived mechanical phenomena.[54] This, combined with a nominalistic view of arithmetic and a constructivist philosophy of mathematical objects, places Barrow most firmly, of the mathematicians considered in this chapter, in the nominalist/constructivist tradition of Gassendi and Hobbes.

According to Barrow, geometry and arithmetic "are not conversant about different Matters, but do both equally demonstrate Properties common to one and the same Subject," namely, the consideration of the nature of magnitude.[55] Whereas arithmetic only considers particular instances of magnitude, however, geometry is able to investigate the properties of magnitude in the abstract, which gives it greater power and scope. Barrow is careful, though, to assure his audience that he is not attempting to remove arithmetic from the mathematical canon; rather, he proposes to "restore it into its lawful Place, as being removed out of its proper Seat, and ingraff and unite it again into its native Geometry, the Stock from whence it has been plucked."[56] Arithmetic, in other words, is simply a branch of geometry: it is the tool for investigating the particular cases for which geometry provides the general rules, and thus (*pace* Wallis) cannot possibly be the foundation for mathematics.

In Barrow's epistemology, 'universal' concepts can be expressed in geometry because the mind forms general ideas of things it experiences in nature and abstracts the properties of sensible phenomena to form geometrical representations. But Barrow notes that mathematical descriptions are both general and particular, depending upon the epistemological level at which a question is considered: mathematical objects "are at the same time both intelligible and sensible in a different respect; intelligible as the Mind apprehends and contemplates their universal Ideas, and sensible as they agree with several particular Subjects occurring to the sense." Barrow does not advocate an epistemology that is either fully universalist or entirely nominalist; rather he believes "there is no reason why the Doctrine of Generals should be separated from the Confederation of Particulars, since the former entirely includes and primarily respects the latter."[57] This, in effect, is one strategy for neutralizing the tension between realist and nominalist epistemologies, but it is not a complete compromise, since Barrow maintains the position that general ideas must still have a foundation in sensible objects that are concrete. Barrow also distinguishes between general 'ideas,' which are the products of experience mediated by the intellect, and the words or symbols used to represent them, since he regards the latter as arbitrary signs whose signification is determined by nothing more than convention.

In practice, this leads Barrow to a reaction that is opposite to Wallis': Barrow sees algebra, which is a language of signs for expressing the general relations between magnitudes abstractly, as a baseless formal system. As he explains:

> *Analysis*, understood as intimating something distinct from the Rules and Propositions of *Geometry* and *Arithmetic*, seems to belong no more to *Mathematics* than to *Physics*, *Ethics*, or any other Science. For this is only a Part or Species of *Logic*, or a certain Manner of using Reason in the Solution of Questions, and the Invention or Probation of Conclusions, which is often made use of in all other Sciences.[58]

It is, in short, defensible to use geometry to speak abstractly about magnitude and arithmetic to speak about particulars, but to combine the two methods to speak abstractly in the grammar of arithmetic deprives mathematics of tangible reference. Wallis, of course, might accept the arbitrary character of analysis that Barrow describes, but unlike Barrow does not see this as an impediment to its usefulness in mathematics, because Wallis is more comfortable with the notion that mathematics is a descriptive formal system. It is, however, important to note that what Barrow is criticizing is the metaphysical or ontological basis for analysis, which he sees as much weaker than that of geometry. Barrow seems, in fact, quite willing to accept analysis as a *practical* measure or tool, provided that its status is clearly recognized as "an Instrument subservient to the Mathematics." Since *Usefulness* is primarily an explication of the philosophical foundations of mathematics, Barrow spends more time dogmatically justifying the status of geometry than he does considering all potentially legitimate mathematical applications; he should be interpreted cautiously here, because it is possible to read his criticism of analysis as more thoroughgoing than it actually is.

Nonetheless, Barrow is quite forceful in his attack on the ontological basis of algebra, which he extends to a more general critique of the foundations of arithmetic based on nominalist epistemological convictions. To put it bluntly, Barrow questions whether numbers have any real existence at all. "I remark," writes Barrow:

> that no Number of itself signifies any thing distinctly, or agrees to any determinate Subject, or certainly demonstrates any thing. For every Number may with equal Right denominate any Quantity: and in like manner any Number may be attributed to every Quantity . . . Whence it appears that no Number can design any thing certain and absolute, but may be applied to any Quantity at Pleasure.[59]

Here Barrow is not just saying that the marks we use to represent numbers are arbitrary; rather, he is claiming that the entire basis of arithmetic is conventional. Or, to put it another way, Barrow is claiming that 'numbers' are nothing more than the symbols we use to represent numerical quantities there is no metaphysical or ontological basis for numbers as objects with real existence. As Barrow explains, "a *Mathematical Number* has no Existence proper to itself, and really distinct from the Magnitude it denominates, but is only a kind of *Note* or *Sign* of Magnitude considered after a certain Manner."[60]

Barrow's justification for his stance is based in large part in his belief that mathematics must represent and describe real objects and phenomena. Again, he has no problem with applying arithmetic to particular cases for example, counting sheep since what is at stake ontologically is clear: if there are five sheep, the object being considered is not the number 'five,' but rather the five individual sheep (whose ontological status is not in question).

The problem comes when arithmetical rules are applied generally, as in the case of algebra or 'specious arithmetick.' Barrow regards arithmetical operations as invalid if the units being quantified cannot be defined (such as $\sqrt{\ }$ 1), since there is nothing which guarantees that the calculation will apply to homogeneous units. Barrow considers the proposition 2+2 = 4 and questions whether it should be considered a universal rule:

> Whence comes it to pass that a Line of two Feet added to a Line of four Feet, four Palms, or four of any Denomination, if it be abstractedly, *i.e.* universally and absolutely true that 2+2 makes 4? . . . from whence I infer that 2+2 makes 4, not from the abstract Reason of the Numbers, but from the Condition of the Matter to which they are applied, *i.e.* because any Magnitude called by the name of *Two* added to a Magnitude whose Denomination is *Four*: Nor indeed can any thing be imagined more absurd than to affirm that the Proportions of Magnitudes to one another depend upon the Habitude of the Numbers whereby they may be expressed.[61]

Barrow, in fact, singles out Wallis here for specific criticism, and suggests that Wallis' inversion of the 'proper' order of geometry and arithmetic is based on the logical fallacy of assuming that universals prove particular cases, rather than the other way around.

Despite his approach to arithmetic, however, Barrow is not a thorough going nominalist. He remarks that "we may take notice of their Opinion who will have Mathematical Figures to have no other Existence in the Nature of Things than in the Mind alone," expressing his surprise that "this Opinion should be embraced by Persons, who are otherwise most excellently skilled in the mathematics."[62] Barrow rejects this position because it would undermine his beliefs about the universality of geometry, which he holds to produce general and invariant demonstrations. These demonstrations are, nonetheless, not without their conventional aspect. As Barrow explains in the *Lectiones geometricae*, the objects and figures of geometry are homogeneous, ultimately reducible to the rectilinear motions of points:

> For every line that lies in a plane can be generated by the motion of a straight line parallel to itself, and the motion of a point along it; every surface by the motion of a plane parallel to itself and the motion of a line in it (that is, any line on a curved surface can be generated by rectilinear motions); in the same way solids, which are generated by surfaces, can be made to depend on rectilinear motions.[63]

Because of the homogeneity of their objects, geometrical demonstrations can be applied to all cases in which magnitudes are produced, and we can fix them with "clear and distinct Ideas, designing them by proper, adequate and invariable Names," which produce conclusions "in the highest Degree consonant to Reason and not to be denied by any one in his right Mind." This kind of reasoning is, Barrow declares, "properly *Demonstrative*, and produces the most certain Knowledge which the Mind of Man is capable of."[64] Unlike arithmetic, whose axioms dictate only the rules for combinatorial relations between given objects (but do not define the objects themselves, which are arbitrarily imposed), the axioms of geometry are applied to objects whose signification is not in doubt, since they are defined by "the Form and inward Constitution of the Thing expressed by the Definition."[65]

Barrow is asserting that we can know the formal cause of geometrical objects with complete assurance, because we are able to consistently define geometrical objects within the axiomatic system of geometry, and here he combines mechanical constructivism with more general mathematical constructivism. In a certain sense, however, a system that produces invariant demonstrations based on objects defined by their formal properties produces only a formal kind of truth and certainty. Barrow does not exempt the names and definitions of geometry from his conventional philosophy of representation: recall his statement that

> the Truth of a Definition is merely arbitrary as to its Subject . . . [and] depends upon the Possibility of the Hypotheses, which it includes, so that to make a Definition true, the Name assigned must belong to a Thing following some possible Supposition expressed in it.[66]

Barrow obviously regards the suppositions expressed in geometrical definitions as highly probable, but he does not believe that geometrical definitions capture essences of the objects they describe. Rather, he hopes at the very least that "Mathematicians do use such Words as make their Definitions perspicuous, and without the least Ambiguity," so as

> [to] express their Conceptions with no more adequate Terms, such as are of a certain and inevitable Signification; so that, as soon as ever the Term for the Thing proposed is heard, the Conception answering to it immediately occurs to the Mind.[67]

Barrow is, in effect, calling for the same precision and consistency in representing geometrical ideas that Gassendi, Hobbes, Locke and others have argued for; it is important to note, however, that the "inevitable Signification" that he cites is signification of ideas in the mind, and not of the native qualities or essences of the objects themselves. This attitude again expresses a kind of compromise between naive realism and outright nominalism: Barrow believes that it is possible for language to capture the essential qualities of ideas in the mind, but recognizes that because ideas are themselves representations, there is no exact correspondence between natural objects and our linguistic (or formal) descriptions of them.

Mathematics and the Physical World

The preceding discussion relates directly to Barrow's beliefs about the ability of mathematics to produce useful knowledge about the physical world. While Barrow is happy to destabilize the ontological foundations of arithmetic, he is more hesitant when it comes to geometry, since he believes that on some level geometrical demonstrations do correspond with physical realities. At the root of any axiomatic, deductive system (such as geometry) are its axioms or first principles that cannot be proven from within the system, and Barrow is agnostic about the ultimate origins of the first principles of geometry:

> by whatsoever Way the certain Knowledge of Axioms is acquired, whether by Induction and manifold experience, or it arises from the natural Notions of the Mind, or whatsoever other Way it is attained; it is sufficient that we are intimately conscious of it, and find it impressed upon the Mind in an indelible Character.[68]

Barrow suspects that sense plays a role in acquiring general principles, but he is unwilling to attribute this solely to observation or induction, since geometry deals with objects such as perfectly straight lines and exact circles that are outside of our experience. He rather suggests that the senses and reason complement one another, and that the intellect performs a function that is analogous to sensory perception in discerning the abstract general properties of an object or figure that the senses alone cannot perceive.[69] Barrow does, however, grant that in verifying mathematical demonstrations, sense is the first faculty to be relied upon:

> where any Proposition is found agreeable to constant Experience, especially where it seems not to be conversant about [concerned with] the Accidents of Things, but pertains to their principal Properties and intimate [inner] Constitution, it will at least be most safe and prudent to yield a ready Assent to it.[70]

A persistent feature in Barrow's epistemology and which we have encountered before in the work of Wallis, Hobbes, Locke and others is the notion that a statement can be 'true' without necessarily describing an ontologically real entity. This is an important element of constructivist epistemology, and I would certainly place Barrow within the general tradition of epistemological constructivism. In his discussion of this subject, however, both in *Lectiones geometricae* and *Usefulness*, Barrow introduces an additional wrinkle which makes comparison to Gassendi and Hobbes more compelling: a combination of general mathematical and epistemological constructivism with theological voluntarism. According to Barrow, objects that can be described by mathematical demonstrations but which do not exist in the actual physical world may also be said to be potentially real, since in having no inconsistencies they could possibly be created by God at any time. Antoni Malet has described this feature of Barrow's philosophy, noting that "Barrow's first principles need not be self evidently true they only must be free from contradiction." According to Malet, mathematical demonstrations may produce logically coherent descriptions of 'worlds' that do not correspond with reality as we perceive it, but rather which "describe 'imaginary' worlds which God might create."[71]

Barrow uses this notion to defend the mathematical coherence of the several different cosmological doctrines that had been advanced over the centuries. He notes that the theories of Ptolemy, Copernicus, Tycho, and Kepler are equally valid from a purely mathematical point of view, though they do not all present an accurate picture of the actual physical world. "I say," writes Barrow:

> though I imagine all these to be false, at least unknown in respect of those Stars; yet because nothing hinders, but God may create such a World, where the Stars will exactly agree with such Motions; therefore the Demonstrations depending upon such Hypotheses are most true, and their Astronomy true, not indeed of this World, but of the other, which is supposed capable of being created by God.[72]

This idea of the potential existence of other worlds that are mathematically consistent is tied, according to Malet, to Barrow's theological voluntarism, which posits that God has the freedom to act in the world to change the order of nature at any time. This subject has been much discussed in recent years, most notably in connection with the atomistic philosophy of Gassendi and Charleton, and with the theological basis for Newton's concepts of force and universal gravitation.[73] It is clear that Barrow subscribes to at least a form of voluntarism, as he holds that "every Action of an *efficient Cause*, as well as its consequent *Effect*, depends upon the *Free Will* and Power of *Almighty God*, who can hinder the Influx and Efficacy of any *Cause* at his Pleasure." Barrow grants that God's intervention may disrupt the normal pattern of cause and effect, using the example of "the necessary Existence of Fire inferred from Ashes or Smoak" to question whether one can doubt "but God can immediately create Ashes and Smoak, or produce it by other Means."[74]

Leaving aside the question of whether Barrow believed God actually intervenes in the causal structure of nature, the potential for such action raises at least a shadow of skepticism about the certainty of human understanding. But rather than take this potential uncertainty to be limiting, Barrow seems to have found it liberating: Barrow can treat mathematical representations as if they were 'real,' without having to take the step of professing ontological realism. Thus, Barrow can argue that "though no such Notions be ever found in the Nature of Things, as Geometricians suppose to be described by *Spiral Lines, Quadrices, Conchoids, Cissoids*, &c.," so long as they are consistent with mathematical rules they may be assumed to be 'virtually' real, since they "follow from such Suppositions, [as] are rightly demonstrated" and therefore could exist if God willed it.[75] In a sense, Barrow is privileging the intellect, but he is not proposing an abstract Platonic realm of ontologically real idealized objects. Rather, he is suggesting that mathematicians are not limited by the boundaries of what is physically possible in the real world when constructing virtual 'alternate realities' that are inhabited by fictional objects. Barrow explicitly maintains the distinction, however, between the real and the merely possible, and does not attach this belief to a doctrine of corresponding levels of reality in an ontological hierarchy. Just as these fictional entities might be created by God in material substance, they are constructed in the mind of the mathematician, since "the Domains of Reason do far exceed the Limits of Nature; the intelligible World is vastly farther extended and more diffusive than the sensible World, and the Understanding contemplates many more Things than the Sense."[76]

Still, as Malet points out, in order for mathematics to contribute to practical natural philosophy, "theories need testing to ensure that they apply to *this* world."[77] The obvious empirical strategy is to proceed by inductive reasoning and to compare mathematical results with data collected from many observations or experiments. But evidence gathered by induction must recognize the caveat that no matter how many particulars may be observed, there is no way to assure that comparisons are complete or that theories can be verified with absolute assurance. Here Barrow's 'many worlds' philosophy provides an interesting exemption from this limitation: rather than assuming that induction must be complete, he requires only that enough observations be made to predict the possibility that a general rule may be posited. Barrow claims that "every Sensation is of Things particular, and from the Existence of a particular Thing, we may conclude that something like it may exist, but not that it actually does exist, as has sometimes been taken notice of before."[78] In other words, Barrow combines the nominalist position that we may only experience particulars with the rationalist belief that general knowledge may be acquired about the natural world. But he avoids radical skepticism by suggesting that general features may be extrapolated from individual observations, while also avoiding essentialism by claiming that such generality is only probable. This is reflected in Barrow's claim that "the Truth of Principles does not solely depend on *Induction*, or a perpetual Observation of Particulars," but rather on a modified empirical method in which "only one Experiment will suffice (provided it be sufficiently clear and indubitable) to establish a true Hypothesis, to form a true Definition; and constitute true Principles."[79] It should be remembered that Barrow's notion of a "hypothesis" does not require it to be absolutely certain in an ontological sense; rather, for Barrow a true hypothesis is merely one that is consistent with the logical rules of argument and geometry, and true principles, as Malet observes, "need not be self evidently true they only must be free from contradiction."[80] By this measure, then, principles may be drawn whether one performs one experiment or a thousand, since one is not necessarily trying to prove with absolute certainty that the principles must apply to this world, but rather only that they might.

This is a new kind of epistemological position: it is moderate in terms of its claims of certainty without falling into skepticism, and it captures the spirit of the experimental method without abandoning the generality and power of rational geometry. It is clear that at least on one level, Barrow was influenced by nominalist and conventionalist beliefs about language and logic, which inform his critique of algebra and arithmetic and lead him, in Hill's words, to consider analysis "a tool or heuristic device" much like Wallis.[81] Hill also maintains Barrow's supposed hostility to analysis has been overstated, a claim I endorse. Barrow questions the ontological foundations of algebra, but he does not entirely reject that method as a practical mode of reasoning in certain circumstances. Barrow's hostility is, in fact, far less than Hobbes's, and Barrow's less dogmatic stance can perhaps shed some light on the extent of the epistemological divide between Wallis and Hobbes. Barrow and Hobbes had similar philosophical beliefs about mathematics: both were wary of the formalism of algebra, both favored what Pycior calls a "sense based geometry," and both advocated mathematical constructivism.[82] But whereas Hobbes's "extreme nominalism" drove him to acrimonious debate with Wallis and prevented him from ever acknowledging even the heuristic value of algebra, Barrow's more moderate epistemology leaves open the possibility of useful formal analytic systems.

Ultimately, what distinguishes Barrow from Wallis is that the former is less comfortable with treating mathematics as a heuristic device than the latter. As is clear from his work on geometrical optics, Barrow was committed to mathematizing natural phenomena, and he wanted his geometry to reflect or represent nature in a 'language' that was as closely analogous to the phenomena of mechanics as possible. Barrow's chosen language was geometry, because (like Hobbes) Barrow thought geometry best suited this task; his epistemological approach is, however, much more accurately described by Blay's term 'mathematization' than it is by 'geometrization,' despite the fact that Barrow favored geometry. Barrow's geometry does not entirely "put aside any meaningful ontological claims," but it stretches the definition of what an ontological claim is to include demonstrations of phenomena that may not exist in this world. God is still the supreme geometer in Barrow's philosophy, but His omnipotence extends from his ability to create, at will, physical structures to match any hypothetical ones posited by the logical rules of geometry. Human beings 'reflect' this divine characteristic not by being endowed with the ability to peer into God's mind (as Descartes suggested), but rather by virtue of their ability to themselves construct even if only in the intellect geometrical 'objects' that do not physically exist. Thus we can see Barrow in his clear constructivism and hesitant nominalism as a bridge between the epistemologies of Gassendi and later seventeenth century mathematicians.[83] In Barrow's work, the tension between realism and nominalism is evident, but he manages (via constructivism) to find a compromise with which he appears comfortable. This tension is also highlighted in the work of the last figure to be considered in this chapter Newton who, unlike Barrow, was not able (even, I argue, to his own satisfaction) to resolve this epistemological dilemma.

Isaac Newton

These themes discussed by Barrow were reflected in the mathematical writings of Newton, which is not surprising given the close personal and intellectual relationship between the two men. In an important way, however, Newton's philosophy also transcended the debate as envisioned by Gassendi and Hobbes. There is an undeniable epistemological tension in Newton's work between his conviction that 'pure' mathematics (specifically geometry) is absolutely true and certain, and his simultaneous awareness that the powers of the human intellect are limited. Again and again, Newton's writings express the extreme confidence he had in his mathematical demonstrations, but he also at times backed away from making absolute claims about the reality of his mathematical demonstrations when applied to the physical world. Newton's mathematical methodology, particularly in the *Principia*, has been much discussed by historians. I. B. Cohen has described what he calls the "Newtonian style," which involves "the possibility of working out the mathematical consequences of assumptions that are related to possible physical conditions, without having to discuss the physical reality of those conditions at the earliest stages."[84] This 'style' relied heavily on modeling nature mathematically, but the final relationship of those models to physical reality remained a sticky issue for Newton. Mahoney suggests that this issue centers on the problem of the relationship between the structure "of the mathematical model and the structure of the physical system it is meant to represent and thereby explain."[85] Newton wanted a genuine correspondence between mathematical models and nature, and, as Dear has suggested, one available route was mathematical constructivism. Newton's constructivism was less broadly epistemological than that of Gassendi, Hobbes, and Barrow, but it nonetheless allowed him to conceive mathematical objects as "things *to be drawn* rather than pre existing in a Platonic realm," and thus provides an avenue for comparison of his beliefs with the goals of the nominalist authors.[86]

Labels such as 'nominalist' and 'realist' break down, however, when trying to describe Newton's approach to some of the most important ontological entities in his system. Whereas realists and nominalists centered much of their debate on the ontological correspondence between concepts in the mind and idealized ‘objects’ (such as circles or triangles) in an extra mental or extra sensory realm, concepts such as Newton’s ‘force’ have no clear analogue in either philosophical tradition. Force, which Newton unquestionably conceived as an ontologically real entity, is not an ‘object’ in the same sense as either the metaphysical essence, universal category, or Platonic form of a triangle is. Neither, however, is it an artificial construction of the mind, a ‘concept’ without a physical referent. Force is a distinctly new class of ontological being: it is real without being tangible, and its effects are mathematically quantifiable but its physical properties are unknown and perhaps unknowable. Although Newton uses geometry (and analysis) to demonstrate the existence of gravity, the force itself cannot be conceptualized as a simple geometrical object or construction, but rather requires the complicated series of geometrical demonstrations in the *Principia*. Newton also conceives force as an immanent property of God’s will, and God’s power, being infinite, cannot be reduced to a simple category or substance. Relating Newton’s work to the concerns of the authors of the preceding decades is complicated by the fact that certain elements in his ontology like his treatment of matter and space do overlap with the conceptions of Gassendi, Hobbes, and Descartes, and that his mathematical epistemology (particularly the constructivist element) has parallels with Barrow’s. It is possible to read a nominalist influence in Newton, because he was indeed influenced by nominalists.

We have a very good idea, from student notebooks kept during the two years before his *annus mirabilis* of 1666, that Newton’s early intellectual influences were wide ranging and diverse. Newton read Walter Charleton on atoms and Gassendist physics, Hobbes’s *De Corpore*, and much of Descartes’ *œuvre*, as well as works by Wallis, William Oughtred, Christoph Clavius, and other important mathematical authors of his time.[87] J. E. McGuire and Martin Tamny argue for a strong atomistic interest in this early period, and suggest parallels between Newton’s speculation about physical atoms and his interest in Wallis’ method of indivisibles. This, McGuire and Tamny propose, reveals Newton’s desire to find a physical basis for the ontologically problematic mathematical entities found in the new analytic methods.[88] They argue furthermore that although Newton was “inclined” towards mathematical realism during this period, “with regard to the quantity of objects that exist extramentally, Newton is an Epicurean actualist.” McGuire and Tamny contend that though the young Newton was very concerned with aligning mathematical representations with physical reality, he was nonetheless a “mathematical conceptualist,” who recognized a divide between concepts of mathematical objects in the mind and objects in the physical world. Furthermore, they argue that this ontological concern led Newton to reject traditional infinitesimals (which he replaced with fluxions) and strongly informed the development of his mature mathematics.[89]

These questions led Newton, throughout his career, to develop a philosophy of mathematics and particularly geometry that was ‘physicalist’; that is to say, a belief that mathematical representations should be closely aligned with the properties of physical bodies and their motions. Newton's fluxional calculus differed from Leibniz's version in that Newton conceived of infinitesimal increments not as infinitely small quantities (which as objects would be difficult to justify ontologically), but rather as succeeding 'moments' observed as a particle moved along a path over time.[90] The path of a curve could be treated as the combination of forces exerted on the particle as it moved, which had obvious advantages when it came to representing a body under the influence of gravity. What is important to note here is that this method of mathematical representation was actually an attempt at an accurate 'description' of a physical event; the method of fluxions describes a narrative 'history' (though a fictional or hypothetical one) of the motion of an object as it traverses a distance over a particular time. That this physicalist conception of mathematical representation was influenced by Hobbes's geometry is evidenced by the careful notes Newton took on *De Corpore* in his *Quaestiones*.[91]

This approach to mathematical representation also accounts for the ambivalence Newton had towards algebraic notation over the course of his career, which deepened to outright hostility as time wore on. As Niccolò Guicciardini explains, Newton turned from an analytical mode of expression to "a geometrical method based on limits," a move Guicciardini calls "one of the most spectacular processes in the history of mathematics," and which he compares to Einstein's rejection of quantum theory on epistemological grounds. Having helped to lay the foundations for the analytic calculus, Newton ultimately "refused to rely on it," and this is one of the reasons Guicciardini cites for Newton's choice of mathematical demonstrations in geometry rather than algebra in the *Principia*.[92] The decision concerns what Guicciardini terms "one of the most important foundational questions faced by seventeenth- and eighteenth century mathematicians," namely, whether mathematical objects are ontologically real. Though many mathematicians answered 'no,' their epistemological justifications were often in total disagreement. Whereas Leibniz accepted that formal symbols, empty of necessary referential content, could yet lead to fruitful demonstrations, Newton contended, as Guicciardini puts it, that "mathematics devoid of referential content could not be acceptable."[93] This is the same issue that Wallis and Barrow struggled with: whereas Newton (like Barrow) was ultimately unsatisfied with a formal or heuristic interpretation of mathematics because of ontological concerns, Leibniz (like Wallis) was more willing to accept the formalized symbolic language of algebra as a helpful device for solving general problems. Both Newton and Barrow wanted mathematics to represent something concrete, and that meant using geometry.[94]

In his unpublished mathematical papers, Newton compares algebra with geometry in several places. Newton states in his 'Final' *Geometriae libri duo*, for example, that he does "not at all approve of the new generation of geometers" who introduce curves into analytic geometry that have not been demonstrated by traditional methods, since the curves are “arbitrary and [have] no foundation in geometry.”[95] His concern here evidently stems from his conviction that Euclidean synthetic geometry reliably represents mechanically constructed curves, and that by accepting curves that have not been constructed in this manner, analytic geometers risk losing the rational rigor of geometry. Newton continues by lamenting that “present day geometers indulge too much in speculation from equations,” which they mistakenly interpret to provide general demonstrations. As Newton contends (adopting the traditional view), the true method for composition should be the synthetic method, and while analysis at best “guides us to the composition,” “true composition is not achieved before it is freed from all its analysis.” In other words, Newton is echoing Barrow’s claim that reliable knowledge cannot be produced by analytical methods, since analysis is suited to the examination of specific problems, not the production of general solutions. “Simplicity in figures,” Newton explains, “is dependent on the simplicity of their genesis and conception, and it is not its equation but its description whether geometrical or mechanical by which a figure is generated and rendered easy to conceive.”[96]

By defending classical methods of geometrical composition, Newton is claiming that “his method [is] more than a mere heuristic tool,” as Guicciardini contends, which suggests that “geometrical interpretability guarantee[s] ontological content.”[97] Newton is not, however, suggesting that the geometrical representations themselves are the ontologically real entities they describe, but rather that their referents are: geometrical representations describe actual physical entities. Does this mean that Newton believes there is a one to one correspondence between geometrical representations and the natural phenomena they describe? He does not give a definite answer to this question. Indeed, it is clear that Newton does not expect geometry to answer metaphysical questions about nature at all; rather, Newton describes it as a method “devised, not for the purposes of bare speculation, but for workaday use,” and that as a “science [it] wins our gratitude in consequence of its usefulness.”[98] Geometry is useful because it can provide certain results, but that certainty is tied to its use of synthetic reasoning, which begins with axioms that are self evident and which are not themselves the subject of investigation and proof.

The need for reconciliation between the deductive certainty of pure geometry and the actual existence of physical entities was necessary in order for Newton’s geometry to satisfy his demand of ‘usefulness.’ This led him to a kind of constructivism, which, in Dear’s words, held that “mathematical knowledge (for which, typically, geometry stands as the exemplar) is about particular kinds of objects created by active construction not contingently, but in their very essence” (meaning that their ‘essences’ are constructed, not found naturally).[99] As we have already seen, one constructivist strategy was to posit that mathematical objects are merely fictions that inhabit a hypothetical world (Gassendi), or more strongly that they exist, if not in this world, then in a potential one (Barrow). Newton demanded more from his mathematical account. In the General Scholium to the *Principia*, Newton writes that "it is enough that gravity really exists and acts according to the laws we have set forth," and there is no indication that he did not conceive of gravitational force as a real entity.[100] Newton does not indicate, however, that geometry explains what gravity is or why it exists, but rather he holds that it describes laws (observed in the quantifiable motions of physical bodies) and demonstrates how gravity acts.

Gravity is not a 'constructed' object according to Newton — it is physically real — but the objects of geometry are constructed in the mechanical sense, meaning that they are conceived as describing the motion (or combination of motions) that might be taken by a physical object in actual space. There is a link between physical description and geometrical demonstration in Newton's epistemology, but geometrical objects are not direct representations of either particular physical entities or of idealized universal forms. Attempting to describe Newton's ontology in those terms is futile: his ontology of force is literally incommensurable (in the Kuhnian sense) with the previous conceptions of both realists and nominalists.[101] This does not mean that Newton's overall natural philosophy is entirely disconnected from either Gassendi's or Descartes', but it does mean that certain concepts cannot be translated between them. What has occupied the previous philosophers in this study has been the perception, conceptualization, and representation of what I have been calling 'objects.' Newton's introduction of force presents a new concept, which I have called an 'entity,' that is not precisely synonymous with the earlier term, and thus moves the discussion of the status of mathematical ontology beyond the parameters of the medieval and early modern nominalist debate. Nonetheless, Newton's contribution to mathematical philosophy merits consideration in regard to that tradition: if it is fair to say that if Newton was not strictly a nominalist (and no one has ever, to my knowledge, argued that he was), then nor was he a 'realist' in the same sense as Kepler or Galileo. The nominalist influence in philosophy of mathematics certainly played a role in informing the development of Newton's early mathematical sensibility (through his readings of Gassendi, Hobbes, and Barrow), and this early influence sheds light on decisions he made in his later mathematical works.

The Calculus Debate

Towards the close of the seventeenth century, natural philosophers were beginning to develop physical models that moved beyond the strict limitations of the Cartesian mechanical philosophy, which considered only body and motion as causal explanations in physics. One of the focal points for this shift in natural philosophy was Newton's introduction of the concept of force into mechanics, which drew praise as well as criticism for harkening back to what some observers termed the 'occult' explanations of Aristotelian and mystical philosophies.[102] Changes in mathematical techniques and practice also led to epistemological and ontological uncertainty in mathematical philosophy. Many practitioners felt compelled, for example, to choose between geometry and the new analytical methods, and (as we have already seen) arguments over this issue heightened concerns about the ultimate ontological reality of different kinds of mathematical objects and demonstrations. Although most scientists were able to comfortably justify the use of mathematics in natural philosophy, when compared to the situation a hundred years earlier there was a striking lack of consensus in the late seventeenth century about exactly *which* mathematics to use and how far to read its conclusions. Overall, the scientific community was in what might not inaccurately be described as a state of Kuhnian 'crisis' over how to proceed with mathematical natural philosophy.[103]

This crisis was felt most deeply by philosophers who were concerned with the metaphysical or ontological foundations of the application of mathematics to physics. Newton's reluctance to present the *Principia* in the language of his calculus of fluxions left the task of defending his analytic methods primarily to his later followers, and the conflict between proponents of the fluxional and infinitesimal calculus eventually led to the messy and well documented clash between Newtonians and Leibnizians in the first decades of the eighteenth century.[104] While historians have often characterized this debate as an example of national and rhetorical conflict, there was undoubtedly a metaphysical and philosophical schism as well. On one level, the arguments simply concerned the technical language for expressing ideas in the calculus, and Leibniz's method of infinitesimals eventually gained acceptance (first on the Continent, later in England) partly because it was praised for being simpler and more elegant than Newton's fluxions. On a deeper level, however, the debate over the calculus was tied to fundamental questions about the nature of mathematical knowledge in general, and about the relationship between mathematics and physical reality. By the end of the seventeenth century the widespread acceptance of analytical techniques and an increasingly formal reading of mathematics had solidified the interpretation of mathematics as a science (or language) of signs, but there was as yet little agreement about the epistemological and ontological status of that science.

The debate between Leibniz and Newton's mouthpiece Samuel Clarke is usually cited as the most significant (and substantial) contemporary challenge to Newton's philosophy, and the hearty support of the Royal Society against the eminent German philosopher is taken to be an example not only of scientific disagreement, but also of national bias. One of Newton's most vocal critics, however, was George Berkeley, whose theory of perception, representation, and mathematics undercut many of the central theses of the mathematical mechanical philosophy of the seventeenth century and was an important dissenting voice against the increasing acceptance of infinitesimal methods and the calculus.

Berkeley's epistemological critique of arithmetic and his metaphysical concerns about geometry are drawn out most explicitly in his attack against proponents of the Newtonian calculus in his 1734 treatise, *The Analyst; or a Discourse Addressed to an Infidel Mathematician. Wherein it is examined whether the Object, Principles and Inferences of the modern Analysis are more distinctly conceived, or more evidently deduced, than Religious Mysteries and Points of Faith.* At stake for Berkeley were three central issues: (1) the epistemological justification for drawing physical hypotheses from a symbolic mathematics; (2) concerns about the ontological status of fluxions; and (3) related to this, questions about mathematical rigor involved in demonstrations that operate with infinitesimal magnitudes.

In the first place, Berkeley was concerned that practitioners of the symbolic analytic method were not sufficiently aware of the arbitrariness of their technique. "It may be justly questioned," Berkeley states, "whether, as other Men in other Inquiries are often deceived by Words or Terms, so they likewise are not wonderfully deceived and deluded by their own peculiar Signs, Symbols, or Species." Berkeley does not regard symbolic analysis as a necessarily illegitimate method, but he cautions that without a proper awareness of what the symbols purport to signify, "much Emptiness, Darkness, and Confusion," and indeed "direct Impossibilities and Contradictions" may pass unnoticed.[105] Analysis is acceptable as a logical exercise, but one should be wary of drawing conclusions about physical nature from it. Berkeley states:

> I consider the Geometrical Analyst only as a logician, i.e., so far forth as he reasons and argues; and his Mathematical Conclusions, not in themselves, but in their Premises; not as true or false, useful or insignificant, but derived from such Principles, and by such Inferences.[106]

According to Douglas Jesseph, Berkeley's approach to arithmetic and algebra was formalist: Berkeley was satisfied with symbolic analysis insofar as it "consist[s] of a vocabulary of primitive symbols, rules for combining these symbols into formulas, a privileged class of formulas . . . and rules for deriving additional formulas from the axioms," but he maintains that "the subject matter of mathematics is the mathematical symbols themselves and not some external realm of objects to which these symbols refer."[107]

However, this comfort did not extend to geometry, and here is where he objects to formalism in the calculus. Calculus purports to explain geometric magnitude, and the Newtonian calculus of fluxions in particular—with its orientation towards the geometry of motion—assumes an analogy with real bodies in real space. Berkeley objects to this analogy, and questions whether reliable results can be obtained when geometry is translated into algebraic form, because it is impossible to maintain the clear ideas of the concepts' original roots in sensory phenomena. Our clearest ideas are of things we have direct (visual) perception of, which is why geometry produces clearer notions than arithmetic; the use of analytic geometry prompts Berkeley to wonder "what Objects you are conversant with, and whether you conceive them clearly."[108] This problem is intensified by the questionable in Berkeley's view ontological status of fluxions, which Berkeley claims cannot be understood geometrically, let alone in symbolic form:

> And what are these Fluxions? The Velocities of evanescent Increments? And what are these same evanescent Increments? They are neither finite Quantities, nor Quantities infinitely small, nor yet nothing. May we not call them Ghosts of departed Quantities?[109]

Berkeley questions whether it is possible at all to have an idea of a fluxion, which he claims is simply an attempt to express infinitesimal quantities without appearing to rely on a fictitious mathematical entity. He regards fluxions to be as ontologically problematic as are Leibnizian infinitesimals, calling all such concepts,

> {whether} momentaneous Increments, nascent and evanescent Quantities, Fluxions and Infinitesimals of all Degrees . . . in truth such shadowy Entities, so difficult to imagine or conceive distinctly, that (to say the least) they cannot be admitted as Principles or Objects of clear and accurate Science.[110]

Berkeley caps off this ontological critique of fluxions by claiming that even if he were to grant that fluxions are reliable concepts, practitioners of the calculus further invalidate their use (and ontological existence) with faulty reasoning. This charge regards the practice of first assuming that an increment of flow (a fluxion) has quantity in an equation, and then dropping it from the result, without affecting the demonstration. In Berkeley's example, he supposes:

> that the Quantity x flows, and by flowing is increased, and its Increment I call o, so that by flowing it becomes $x+o$. And as x increaseth, it follows that every Power of x is likewise increased in due Proportion. Therefore as x becomes $x+o$, x^n will become $(x+o)^n$.[111]

The problem is that in the end, the incremental value o is allowed to vanish, but the values for x which were obtained using o are kept. As Berkeley notes, "certainly when we suppose the Increments to vanish, we must suppose their Proportions, their Expressions, and every thing else derived from the Supposition of their Existence to vanish with them," or else we risk a logical fallacy. Berkeley is quite firm on this matter, and refuses to let it drop:

> I repeat it again: You are at liberty to make any possible Supposition: And you may destroy one Supposition by another: But then you may

> not retain the Consequences, or any part of the Consequences of your first Supposition so destroyed. I admit that Signs may be made to denote either any thing or nothing . . . But then which of these soever you make it signify, you must argue consistently with such its Signification, and not proceed upon a double Meaning: Which to do were a manifest Sophism.[112]

In essence, Berkeley is charging that this procedure violates the fundamental principle of algebra: rectification, or the balancing of an equation by subtracting equal quantities from both sides. This is a penetrating criticism and, as several historians have noted, one of the most damaging challenges presented to the logical foundations of Newton's calculus.[113] The above quotation aptly summarizes a number of crucial objections, which ultimately reduce to the charge that fluxions are a logical impossibility.[114] To Berkeley, it is inconsequential that the calculus 'works' despite its logical flaws, since in his mind the same results could be obtained using the more legitimate rules of traditional geometry. That the calculus arrives at the 'truth' is a mere accident, and says nothing about the rigor of its method; in fact, it says rather the opposite, since it is only "by virtue of a twofold mistake" (i.e. a double cancellation) that a correct result is obtained, and "Science it cannot be called, when you proceed blindfolded, and arrive at the Truth not knowing how or by what means."[115]

While the historian Geoffry Cantor has claimed that the major epistemological issues in Berkeley's philosophy of "sensationalism, immaterialism, [and] the critique of abstract ideas . . . are all of indirect relevance to *The Analyst*," preferring theology as the more important influence, I would argue that these issues which are the foundations of Berkeley's nominalist and constructivist approach to language and mathematics are, in fact, central.[116] On this score, Jesseph agrees, finding Cantor's "treatment rather one sided," noting that "the most interesting theses in *The Analyst* are completely independent of theological considerations."[117] Indeed, historians who have dealt explicitly with the place of Berkeley's mathematics in the eighteenth century debate over the calculus have tended to agree that Berkeley's criticisms were penetrating and influential, and particularly have drawn attention to his attack on the logical foundation of fluxions.[118] Berkeley's critique of the ontological and epistemological status of the fluxional calculus is directly dependent upon his rejection of abstract ideas (a 'fluxion' being one such idea), and on his characterization of mathematics as a language of arbitrary signs bound by rules determined by convention.

Ultimately, Berkeley's views directed his conception of the method and subject matter of physics towards a constructivist and somewhat skeptical position. In *De Motu*, Berkeley criticizes mechanical philosophy for unrealistically focusing on determining cause and effect in natural phenomena, which leads to a misguided emphasis on efficient cause:

> it is the concern of the physicist or mechanician to consider only the rules, not the efficient causes, of impulse and attraction, and, in a word, to set out the laws of motion: and from the established laws to assign the solution of a particular phenomenon, but not an efficient cause.[119]

Berkeley's concept of a physical 'law' is again derived from his philosophy of representation: what we consider a law is a conventional, arbitrary relationship constructed from a number of particular observed natural phenomena, and is expressible as a 'rule' that governs the linguistic (or mathematical) formation of statements about those phenomena. A law is not a universal truth, and Berkeley several times refers to Newton's law of gravity as "a mathematical hypothesis."[120] Above all, Berkeley characterizes mechanics or physics as a science of approximation and denies that physics provides any insight into 'reality.' He writes:

> Just as geometers for the sake of their discipline contrive many things which they themselves can neither describe, nor find in the nature of things, for just the same reason the mechanician employs certain abstract and general words, and imagines in bodies force, action, attraction, solicitation, &c . . . even if in the very truth of things and in bodies actually existing they are sought in vain.[121]

Anything we postulate beyond the immediately perceivable sensual qualities in bodies "should be judged to be of the same sort as other hypotheses and mathematical abstractions," since "in mechanics . . . mathematical ideas rather than the true nature of things are regarded."[122]

Jesseph has characterized Berkeley's philosophy of arithmetic as formalist, and he describes Berkeley's account of geometry as a weak version of instrumentalism.[123] I prefer, however, to use the term 'constructivist,' which I think more accurately captures Berkeley's epistemology. As we have seen, Berkeley's general theory of perception and representation prevent him from drawing necessary connections between mathematics and the physical world, even in the case of geometry, whose objects are representations of sensory experiences. In Berkeley's scheme, geometry would have an epistemological edge over arithmetic, since the objects of geometry at least have referents that are not empty signs (unlike arithmetic, whose objects are empty and arbitrary signs), but since he does not believe that we have transparent access to visual data, even objects based on experience are in a sense constructed by the mind. Berkeley's attitude towards both the symbolic and concrete branches of mathematics supports Pycior's contention that Berkeley's philosophy of mathematics reflected the influence of both Wallis and Barrow. Like Wallis, Berkeley was comfortable with a formalist reading of algebra and arithmetic, and he was able to incorporate Wallis' enthusiasm for symbols with Barrow's constructivist skepticism into a philosophy that was positive towards symbolism, but ambivalent about the ontological status of arithmetical and algebraic objects.

In doing so, Berkeley was able to overcome the reservations many of the nominalists before him had expressed concerning algebra including Hobbes, Barrow, and even Newton because he was comfortable accepting formalism without regarding formal objects as ontologically real. Arithmetic and algebra, being purely formal systems, are equivalent to the 'artificial' languages constructed by human beings (e.g. English, German, French, etc.), since their structure is logical and any relation they have to the phenomenal world is assigned entirely by human convention. While geometry, on the other hand, is not a direct representation of physical reality, it nonetheless derives its objects from visual experience and is therefore not entirely arbitrary. This corresponds with Berkeley's notion of the 'visual language of nature,' which geometry might be said to translate into human signs. As Berkeley makes clear in the *Theory of Vision*, the distinction between 'artificial' and 'natural' language rests solely on the authorship of the languages (human vs. divine) and not on any essential difference between their objects or rules.[124] Thus, geometry is, in a very weak sense, the 'language of nature,' since the objects of geometry are drawn from visual perceptions of the 'language' of objects God has placed on earth, and their ordering is based on divine rather than human convention. We do not, however, translate essences with this language or have direct access to its objects, nor does it provide insight into the necessary structure of the universe or the nature of God's mind.

Berkeley's critique of Newtonian calculus did not prompt a wholesale change in contemporary mathematical practice, but it was certainly taken seriously by contemporaries, read and debated, and in a few noteworthy cases actually did inspire an attempt at reforming some of the problematic concepts in the method of fluxions. Pycior and Jesseph note that the mathematical reaction to *The Analyst* was "intense," but much of the debate that followed its publication was focused either on the theological charges in Berkeley's tract (his accusation of "infidelity" aimed at British mathematicians) or did not answer his central mathematical critiques.[125] The most public and violent reaction to *The Analyst* was the physician James Jurin's anonymous response, *Geometry No Friend to Infidelity*, which ostensibly defended both Newton's methods and his theological purity.[126] Jurin's major epistemological point was to claim, *contra* Berkeley, that one could have concrete ideas of fluxions, and his attack prompted a response from Berkeley, *A Defence of Free thinking in Mathematics*, which reiterated many of the major claims already made in the *Principles* and *The Analyst*.[127] In general, Berkeley seems not to have found much favor with contemporary British mathematicians, who by and large stuck to Locke's account of ideas and what they interpreted as Newton's mathematical method. As Pycior notes, "by the 1730s English mathematical thinkers were unprepared for an abstract theory of arithmetic and algebra as sciences of signs," or a conception of geometry which did away with abstract general ideas.[128]

As has been very well documented, the first half of the eighteenth century was a crucial time for the defense and popularization of Newtonian science, which led eventually to the codification of Newton's so called 'mathematical method' and his canonization as a scientific icon. There is a real question about whether what came to be known in the Enlightenment as 'Newtonianism' accurately reflects the philosophy of Newton himself; there were many legitimate, divergent forms of Newtonianism, and it is beyond the scope of this study to examine them in detail. Nonetheless, there was a concerted effort by British natural philosophers to popularize as well as to define the 'message' of Newton's science from the 1720s onward, and Berkeley's critique deserves to be considered a part of this forging of Newtonianism. Works by Henry Pemberton, David Gregory, Samuel Clarke, and Colin Maclaurin ranged from fairly basic explications of the *Principia* for the general reader (such as Pemberton's *A View of Sir Isaac Newton's Philosophy*) to more sophisticated analyses of Newton's theory (including Gregory's *The Elements of Physical and Geometrical Astronomy*), but overall did not attempt to modify Newton's approach or answer any of the criticisms posed by Berkeley.[129]

An exception to this pattern was Colin Maclaurin's two volume work, *A Treatise of Fluxions* (1742), which defended Newton's mathematics while at the same time acknowledging Berkeley's critique.[130] Maclaurin was a Scottish mathematician and eventual professor of mathematics at Edinburgh University who was an extremely well regarded practitioner himself, and his *Fluxions* is regarded as one of the most important works on the calculus of its day. He also (posthumously) published a more accessible survey of Newton's philosophy entitled *An Account of Sir Isaac Newton's Philosophical Discoveries* in 1748 that, although aimed at a more general audience, is remarkable for its explicit attack on Cartesian ontology and cosmology. Taken together, these two works provide an intriguing insight into the early formation of the image of Newton's practice and beliefs and demonstrate that even as formal techniques were more and more generally accepted for expressing the calculus, they still posed some philosophical considerations that troubled mathematicians of the mid eighteenth century. More importantly, Maclaurin's *Fluxions* begins with a direct acknowledgement of Berkeley's *Analyst* and makes a remarkably thorough attempt to deal with the charges in the *Analyst* in a conciliatory manner.[131]

Maclaurin begins *Fluxions* by acknowledging that *The Analyst* "first gave rise to the ensuing treatise," explaining further that Berkeley's attack on symbolism "determined me, immediately after that Piece came into my hands ... to attempt to deduce those Elements after the manner of the Antients, from a few unexceptionable principles, by Demonstrations of the strictest form."[132] Maclaurin in other words believed in the essential basis of Newton's method of fluxions, but recognized that the only way to answer

Berkeley's charges sufficiently was to present the method in the language of classical geometry—not coincidentally following the same approach used by Newton. This is not quite the same thing as saying that Maclaurin rejected algebra; rather, in cases of dispute, he writes, "the most effectual way to set the truth in a full light . . . is to deduce it from axioms or first principles of unexceptionable evidence, by demonstrations of the strictest kind, after the manner of the antient geometricians."[133] This is in line with the traditional belief that geometry is the 'method of demonstration,' which, by virtue of its clear definitions and rigorous logical methods, has an advantage over analysis, which is better suited to discovery. Obviously this strategy assumes that concepts in analysis can be grounded in synthetic demonstration, to which Hobbes, Barrow, and Berkeley would likely have objected, but it does suggest that Maclaurin accepted an epistemological distinction between analytical and synthetic methods in mathematics. Moreover, Maclaurin's willingness to answer Berkeley's criticisms on Berkeley's terms demonstrates that even by the mid eighteenth century, analysis was not so universally accepted that criticisms of its foundations were simply dismissed by every mathematical practitioner.

On the question of the real existence of infinitesimals, Maclaurin seems to sympathize with Berkeley's discomfort, stating that infinitesimals have led geometry from clear principles into 'mystery,' and agreeing that as "the infinites and infinitesimals passed into philosophy," they brought along "the obscurity and perplexity that cannot fail to accompany them."[134] He notes, however, that although many mathematicians have made free use of infinitesimals in the calculus, they have not all "agreed as to the truth and reality they would ascribe to them," and in particular he cites Leibniz, whom he claims conceives infinitesimals as "no more than fictions."[135] In essence, Maclaurin seems to be saying that, while he agrees with Berkeley's underlying ontological problem with infinitesimal magnitudes, he believes nonetheless that synthetic rigor can be established for mathematical entities described by calculus, although he cautions against misuse of such questionable objects. In analysis, therefore, infinitesimals "may be of use, when employed with caution, for abridging computations in the investigation of theorems, or even for providing them when scrupulous exactness is not required." Ultimately, however, since geometry describes only those qualities "such as may be easily conceived to have a real existence," it should avoid infinites, "of which our ideas are so imperfect."[136] Maclaurin exempts Newton's method from this stricture, though, since he does not consider fluxions to be infinitesimals: "I have always represented Fluxions of all Orders by finite Quantities, the Supposition of an infinitely little magnitude being too bold a *Postulatum* for such a science as Geometry."[137] Thus Newton's method of fluxions has an advantage over the infinitesimal calculus, since "the computations in this method are the same as in the method of infinitesimals; but it is founded on accurate principles, agreeable to the antient geometry."[138]

Geometry, according to Maclaurin, deals in its most rigorous form only with objects (or abstracted qualities of objects) that can be 'easily conceived,' but this does not mean that geometry provides knowledge of the absolute natures or essences of those objects:

> The mathematical sciences treat of the relations of quantities to each other, and of all their affections that can be subjected to rule or pleasure . . . We enquire into the relations of things, rather than their inward essences, in these sciences. Because we may have a clear conception of that which is the foundation of a relation, without having a perfect or adequate idea of the thing it is attributed to [Locke, *Essay*, Book II, ii, 25, §8], our ideas of relations are often clearer and more distinct than of the things to which they belong; and to this we may ascribe, in some measure, the peculiar evidence of the mathematics. It is not necessary that the objects of the speculative parts should be actually described, or exist without the mind; but it is essential, that their relations should be clearly conceived, and evidently deduced: and it is useful, that we should chiefly consider such as correspond with those of external objects, and may serve to promote our knowledge of nature.[139]

Like Berkeley and Barrow before him, Maclaurin conceives the object of mathematics to be 'relations,' rather than some actual entities that exist in an ontological sense, because it is easier to have clear ideas of these relations than of the essences of objects that exist in the physical world. This last point resonates with Berkeley's own treatment of mathematical representation, which posits that mathematical knowledge (and indeed all knowledge) is knowledge of the relations that order the logical connections between ideas, based on the associations we draw between perceived objects. Maclaurin is, of course, not nearly so concerned with explaining the perceptual and cognitive mechanisms that underwrite this epistemology, but the above passage does justify to a certain extent Jesseph's attribution of a 'Berkeleyan basis' to Maclaurin's philosophy of mathematics. Where Maclaurin diverges from Berkeley, however, is in his notion of the 'clear and distinct' ideas we draw from mathematics, which is far less pessimistic than Berkeley's and closer to Locke's position. This allows Maclaurin to expect more from arithmetic and algebra, since he believes that "we have no ideas more clear and distinct than those of numbers, and often acquire more satisfactory and distinct knowledge from computations than from constructions."[140]

While Maclaurin may lean towards Lockean abstractionist tendencies, he is still much closer to Berkeley and the nominalists on the ultimate status of mathematical epistemology than he is to Cartesian rationalism. In his much more general and introductory *Account of Sir Isaac Newton's Philosophical Discoveries*, Maclaurin roundly criticizes Descartes on a number of epistemological grounds, which he claims Newton's system corrects. Many of Maclaurin's criticisms are familiar, such as his refusal to admit "objections against plain experience from metaphysical considerations," and his admonishment against "systems that are the product of genius and invention only," which are in fact reassertions of Newton's own statements in the mathematical papers and the *Principia*.[141] He depicts Newton as a philosopher dedicated to induction and experimental method and characterizes geometry as a method of organizing and representing data first collected through experience. Also in a move that evokes Wallis as well he stresses Newton's claim that the proper method of mathematics is to proceed from analysis to synthesis, performing a kind of mathematical 'induction.'[142]

In this work, Maclaurin clarifies his attitude towards abstraction, which he deems unacceptable if not drawn from a careful observation of particulars, again following Locke. "Others," he writes:

> have pretended to explain the whole constitution of things by what they call clear ideas, and by mere abstracted speculations. They express a contempt for that knowledge of causes which is derived from the contemplation of their effects [here citing Descartes], and are unwilling to condescend to any other science than that of effects from their causes. Therefore they set out from the *first cause*; and from their ideas of him [God] pretend to unfold the whole chain, and to trace a complete scheme of his works. This is the philosophy that stands in opposition to our author's [Newton's] to this day.[143]

Neither mathematics nor natural philosophy, in other words, allow us to use deductive techniques to derive the nature of God or the plan of creation, or "to attempt to take in at once, in one great view, the whole scheme of nature," which task "far surpasses the reach of men."[144] It is particularly this use of 'abstracted notions' to attempt to deduce first truths about God and nature that is to be avoided at all costs, although Maclaurin notes that it was a persistent feature in earlier philosophical systems, including Kepler's, which are otherwise to be admired.[145] Maclaurin also backs off from his earlier support of algebra in *Fluxions*, stating that "truth is to be discovered by experiment and observation, with the aid of Geometry, only," and that it is only "by the principles of geometry and mechanics, [that] we are enabled to carry on the *analysis* from the phenomena to the powers or causes that produce them."[146] Maclaurin has thus described a version of Newton that was to remain popular in Britain for the next two hundred years: staunch opponent of Descartes, humble experimentalist, and judicious employer of mathematics only as the tool of induction. Whether this characterization is faithful to the 'real' Newton is a matter for argument, but Maclaurin's Newton certainly does reflect many of the concerns Newton put in writing over the course of his career, and those concerns are by and large the ones I have used to connect him with the nominalist tradition.

Maclaurin's work is also indicative of the state of mind of at least a certain segment of the eighteenth century British mathematical community, which was not entirely hostile to Berkeley's critique of mathematical philosophy and practice, and which, if not itself nominalist or constructivist, certainly showed the influence of that tradition. In his answer to Berkeley's criticisms and in his presentation of Newton's philosophy, Maclaurin fuses Newtonian mathematical method with an epistemology composed of several important elements from the nominalist tradition: Hobbes's, Barrow's and Berkeley's privileging of geometry over arithmetic and algebra, a limited acceptance of abstract ideas drawn from particulars (Locke), a denial of the ontological existence of mathematical objects, an emphasis on the 'relational' aspect and mind dependency of mathematical knowledge, and a pessimism about the availability of knowledge of essences or of the ultimate structure of the universe. Maclaurin's comments about the philosophical basis of mathematics demonstrate that the concerns initially expressed by Gassendi a hundred years earlier persisted in the mathematical culture of the early Enlightenment, and that they to some degree informed the construction of the method and scope of modern mathematical philosophy.

Ultimately, Newton marked the end of a particular era in natural philosophy. Though Newton's name was used to herald the beginnings of a new ('modern') approach to mathematical physics, eighteenth century 'Newtonian' science rarely took the form or epistemological stance of Newton's own method. Mathematics itself became an autonomous discipline during the Enlightenment, and the central philosophical questions about the ontological status and certainty of mathematics were less and less frequently mentioned in mathematical discourse. This is not necessarily because ontological questions were regarded as being 'settled' in the philosophical community; rather, as the scope and interest in formal analytical techniques grew, it was easier to simply accept the heuristic value of mathematics without troubling over metaphysics. Ironically, Newton's own concerns about the validity of formalism came to nothing: Newton's work in the *Principia* proved to be eminently expressible in the Leibnizian symbolic calculus, and though he was credited with co founding the method, Newton's epistemological reservations were neatly excised from the 'Newtonian' achievement that was celebrated throughout Europe. The movement that bore his name often had very scant resemblance to the actual beliefs of the man himself.

The tradition of nominalist and empirical skepticism did not, however, die out in natural philosophy, and particularly in Britain it flourished in the writings of Berkeley and David Hume. As we have seen, Berkeley became a sharp critic of what he viewed as ontological excesses in applications of the calculus, and his is really the final voice in this tradition of epistemological skepticism in logic and mathematics that began a century earlier with Gassendi. But Berkeley's main charge seems to be that mathematicians expect too much of symbolic reasoning; he is in fact comfortable with a heuristic approach to mathematics, so long as it is recognized as being a purely formal method. In his several writings on the subject, Berkeley effectively sums up many of the positions and concerns of Gassendi and Hobbes, and he offers a sophisticated solution to the problem of resolving the tension between formal and concrete views of mathematics that troubled Wallis, Barrow and Newton. Berkeley rejects the notion that ideas in the mind can be abstracted from sense perceptions, and in so doing takes one of the major claims of the nominalists to a radical extreme: namely that mathematics cannot offer descriptions of real objects in the physical world. He also explicitly returns the conversation to the original issue of the parallels between language and mathematics, and in doing so recovers the linguistic heart of the nominalist critique of mathematical reasoning.

In general, the central issues in the epistemological debate between nominalist/constructivists and realist/rationalists extended well into the eighteenth century. Much of the original context of this debate, however its classical and medieval roots, relationship to grammatical and linguistic traditions, connection with atomism, etc. had by the early Enlightenment dropped away. What remained, as its core, was split more and more into two distinct philosophical categories: questions about foundational issues in mathematics, which were increasingly addressed by a specialist audience (and in a more explicitly mathematical idiom) in the later eighteenth and early nineteenth centuries, and more general problems in epistemology, which were reworked by most of the major Western philosophers from Immanuel Kant onward. The debate between realists and nominalists was, I argue, tremendously important to the development of mathematical natural philosophy in the seventeenth century, but it was also a uniquely 'early modern' conflict, and as such did not survive into the modern era. While its seventeenth century proponents often touted slogans of the 'new' science induction, empiricism, physico mathematics they were also steeped in the concerns of the 'old' science scholastic logic, humanism, classical erudition which were of significantly less concern to scientific practitioners of the eighteenth and nineteenth centuries. This is, perhaps, why nominalism in particular has received little attention from historians of science: although it sparked briefly at the center of a crucial debate, its direct relevance lasted little more than a century, after which time most philosophers and scientists could scarcely have been conscious of the sources and motivations of its original proponents.

Conclusion: Mathematization and the Nature of Language

Throughout this book I have maintained a consistent focus on what I have termed the 'nominalist/constructivist' axis of belief represented in the works of Gassendi, Hobbes, Locke, and Berkeley, and I have argued for the influence of this tradition on the mathematical philosophy of Wallis, Barrow, and Newton. The nominalist portion of this axis, I have argued, developed out of explicit and general concerns about the nature of language its epistemological status and the ontological foundation of its objects which contributed to a general critique of Aristotelian and scholastic linguistic and logical theory. This was, for example, the primary motivation of Gassendi, whom I have identified as the originator of the seventeenth century nominalist tradition, and whose positioning between the worldviews of Renaissance humanism and empirical mechanism helped to define the parameters of his critique.

Nominalism, strictly speaking, involves a belief that all knowledge is derived from singular experiences, and implies a rejection of Aristotelian universal categories or Platonic forms. However, the nominalist position of Gassendi (and also Hobbes, Locke, and Berkeley) promoted an additional epistemological convention: the belief that while general knowledge is available, such knowledge is 'constructed' by the mind out of discrete experiences, and therefore does not reflect the necessary ontological reality of the objects it describes. This position I have termed 'constructivism,' or 'epistemological constructivism.' The mind, according to this nominalist/constructivist position, draws data from sense experiences, but it forms general concepts (or ideas) by combining this data in a process of mental construction, and the words we use to signify ideas denote not 'real' objects but rather the mental constructions themselves. A constructivist view of language, then, explicitly maintains an epistemological disconnect between language and 'objective' reality.

While interesting in itself, the greater significance, I have argued, of this 'linguistic constructivism' is that it was consistently applied to the philosophy of mathematics, such that mathematics was conceived as mirroring not nature itself (as many contemporary natural philosophers believed) but rather the structure of language. This relates to the crucial development of strategies for the 'mathematization of nature' in the Scientific Revolution, a concept which, although historiographically problematic, has been a persistent feature of historical explanations of early modern scientific practice. On the view of Gassendi *et al.*, mathematization was underwritten by a notion of construction drawn explicitly from a nominalistic and constructivist theory of language, and mathematics was regarded as a 'language' for describing nature that was subject to the same epistemological conventions that govern the structure, objects, and claims to knowledge of natural languages. This view which might be described as an early form of 'structuralism' represented a departure from the competing notion that physical reality is inherently mathematical, and that mathematics is an *a priori* tool for capturing nature's essence. This distinction may be summarized by noting the difference between conceiving mathematics as the 'language of nature' (the Keplerian/Galilean view) versus mathematics as a 'language describing nature.' The contingently descriptive, rather than ontologically necessary, relationship of mathematics to the physical world characterized an important aspect of the nominalist epistemology, and this critique, I argue, permeated well beyond the limited circle of nominalist authors.

An important tradition in later seventeenth century mathematics held, I (and other historians) have argued, a constructivist view of mathematics that has important similarities to the nominalist tradition. Mathematical constructivism is a more limited position than epistemological or linguistic constructivism, maintaining (as a broad definition) only that mathematical objects are created by the mathematician and do not necessarily represent real objects in physical nature. This mathematical strategy allows 'truthful' demonstrations to be made about otherwise 'fictional' entities (irrational numbers, for example) without invalidating the epistemological claims made by mathematics. On a deeper level, however, mathematical constructivism often shared a common aim with nominalism (and linguistic constructivism): both provided a way to speak about general or universal classes of objects (such as geometrical objects) that avoided making strong claims about the ontological existence of those objects. They both avoided, in other words, an equation of mental or mathematical objects with Aristotelian real universals or with Platonic forms.

Mathematical constructivism, as I have noted throughout this study, has been documented and analyzed by a number of historians, and its centrality in the mathematical philosophies of important figures, such as Wallis and Barrow, has been stressed. In previous historical accounts, however, almost no attempt has been made to correlate mathematical constructivism with nominalism. The aim of this book has been partly to correct that omission. While I have not argued that Wallis, Barrow, and other mathematical constructivists were strict nominalists, the close association of their ideas with nominalist positions suggests that similarity in the epistemologies of these two groups is more than a simple coincidence. Nominalism provided a broad and general epistemological justification for constructivism of all kinds, and the mathematicians I have discussed were certainly familiar with (and in some cases distinctly influenced by) the more general works of important nominalists.

Beyond drawing historical connections between nominalism and mathematical constructivism, however, a deeper goal of this book has been simply to highlight the centrality of mathematics in the general nominalist critique. In studies of the history of mathematics, nominalism is almost never mentioned, and authors such as Gassendi have been ignored as mathematical philosophers. As I have argued, this view minimizes the contribution of nominalism to seventeenth century mathematical philosophy, and provides an incomplete picture of the diversity of viewpoints that contributed to the foundations of mathematics and the application of mathematics to nature during that period. Nominalist authors in particular by participating actively in exchanges with leading natural philosophers such as Descartes and Newton contributed to a lively and very relevant debate concerning the epistemological and ontological foundations of mathematics that was instrumental in shaping general philosophical opinion concerning mathematics and mathematization in the seventeenth century. By offering an antirealist account of mathematical demonstration, nominalists joined a broader tradition of mathematical skepticism that has classical roots, as well as a precedent in the sixteenth century tradition of *Quaestro de certitudine mathematicarum* of Piccolomini and Vieta's symbolic algebra.[1] But although it drew on these older traditions, seventeenth century nominalism, by virtue of its proponents' activities in areas as diverse as matter theory, theology, empirical research, scientific societies, political philosophy, etc., was articulated as a critique central to the concerns and motivations of practitioners of what has been defined by historians and contemporaries alike as the 'new science.' The nominalist position was also distinguished from previous traditions in that it did not attempt to deny the usefulness or certainty of mathematics, but rather questioned the ontological basis for all linguistic and representative mental operations. Here constructivism allowed for mathematics to make meaningful statements without necessitating a philosophy of mathematics that was ontologically realistic.

The very diversity of the early modern period resists easy characterization, and I do not suggest that my study of nominalism should conclude by establishing a fixed category of belief. There were, as recent studies have stressed, multiple mechanical and corpuscular philosophies, multiple versions of empiricism, many forms of 'rationalism,' multiple notions of experiment, and many definitions of knowledge in the seventeenth century, and all are part of what we refer to as the 'Scientific Revolution.' There were also multiple approaches to the philosophy and practice of mathematics, and although I have often opposed nominalism and realism in this study, I am aware that these designations fail to capture many of the viewpoints of that time. I do not propose, therefore, to argue that the nominalist/constructivist mathematical philosophy represents the 'most significant' tradition of the seventeenth century, or to suggest that we need to revise some scale of historical importance to correct for its 'true' impact. Rather, avoiding any such ahistorical judgements, I make the more modest claim that an account of the history of early modern mathematical belief and of the philosophical basis for 'mathematization' in particular that fails to include the nominalists presents an incomplete picture of the complexity and nuance of a very important period in the history of science.

What we can conclude from this study will, hopefully, help to construct a more nuanced appreciation of mathematical philosophy during the Scientific Revolution. By focusing on the linguistic component of nominalist philosophy, I hope to have contributed to an alternative reading of the famous trope that describes mathematics as a 'language of nature.' As stated in the Introduction, when analyzing the historical context of that statement, it is important to consider not only the contemporary meanings of mathematics, but also those of language as well. As I hope this book has demonstrated, different approaches to the question of what constituted language and hence the act of 'reading' produced different interpretations of the mathematical 'text' of nature. In a nominalist/constructivist perspective, I have argued, mathematics provided a language for constructing a text about nature that was a product of human, rather than divine, agency. This directly influenced approaches to the mathematization of nature, suggesting that mathematics is a heuristic tool to be applied to the study of natural phenomena that aids in elucidating nature but does not 'decode' it. This conception of mathematics had broad influence and applicability throughout the Enlightenment and beyond, and in order to appreciate its wide popularity, the contribution of seventeenth century nominalism must be considered and appreciated.

Notes

Introduction

1. There was a pervasive—and some would say pernicious—tone of positivism among early twentieth-century historians of science. Most notable are Ernst Mach, *The Science of Mechanics: A Critical and Historical Account of Its Development* (Chicago: Open Court, 1893), and Pierre Duhem, *Le système du monde: Histoire des doctrines cosmologiques de Plaron à Copernic* (Paris: Hermann, 1913–1959). While perhaps less strident than these earlier authors, a number of subsequent historical accounts emphasized the positive 'accumulation' of scientific knowledge, and ignored possible discontinuities or breaks in historical 'progress': see George Sarton, *A History of Science* (Cambridge, MA: Harvard University Press, 1959), A. Rupert Hall, *The Scientific Revolution, 1500 1800: The Formation of the Modern Scientific Attitude* (London: Longmans, 1954), Herbert Butterfield, *The Origins of Modern Science, 1300 1800* (London: Bell, 1957).
2. For example, Alexandre Koyré, *From the Closed World to the Infinite Universe* (Baltimore. MD: Johns Hopkins University Press, 1957), and E. J. Dijksterhuis *The Mechanization of the World Picture*, trans. C. Dikshoorn (Oxford: Oxford University Press, 1961).
3. Galileo Galilei, *The Assayer* (1623), trans. Stillman Drake in *Discoveries and Opinions of Galileo* (New York: Doubleday Anchor, 1957), pp. 237–238.
4. This is primarily the opinion of E. A. Burtt and Alexandre Koyré, and especially relates to the latter's *Galileo Studies*, trans. John Mepham (Atlantic Highlands, NJ: Humanities Press, 1978); originally published *Études galiléennes* (Paris: Librarie Scientifique Hermann et Cie, 1939), pp. 65–109. All further references will be to the 1978 translation. See also E. A. Burtt, *The Metaphysical Foundations of Modern Science*, 2nd edn (revised edn. Atlantic Highlands, NJ: Humanities Press, 1952; original 1924).
5. Thomas M. Lennon, *The Battle of the Gods and Giants: The Legacies of Descartes and Gassendi, 1655 1715* (Princeton, NJ: Princeton University Press, 1993), p. 239.
6. On the relationship between nominalism and early modern thought, see Louis Dupre, *Passage to Modernity: An Essay on the Hermeneutics of Nature and Culture* (New Haven, CT: Yale University Press, 1993), especially Chapters 3, 5, and 7, and Amos Funkenstein, *Theology and the Scientific Imagination from the Middle Ages to the Seventeenth Century* (Princeton, NJ: Princeton University Press, 1986). For a discussion of the relationship between medieval and early modern theology and nominalism, see Margaret J. Osler, *Divine Will and the Mechanical Philosophy: Gassendi and Descartes on Contingency and Necessity in the Created World* (Cambridge: Cambridge University Press, 1994), pp. 115–116. On nominalism and the history of biology, see David N. Stamos, "Was Darwin Really a Species Nominalist?", *Journal of the History of Biology* 29 (1996) 127–144, and Ernst Mayr, *Evolution and the Diversity of Life* (Cambridge, MA: Harvard University Press, 1976), pp. 429–430.

7 Douglas M. Jesseph takes up the subject of Thomas Hobbes's nominalism in *Squaring the Circle: The War between Hobbes and Wallis* (Chicago: University of Chicago Press, 1999), pp. 205–211, and he also discusses nominalism in relation to George Berkeley's formalist mathematics in *Berkeley's Philosophy of Mathematics* (Chicago: University of Chicago Press, 1993), pp. 118–120.

8 H. Floris Cohen, *The Scientific Revolution: A Historiographic Inquiry* (Chicago: University of Chicago Press, 1994), Chapter 2. For recent revisionist scholarship on the development of the mathematical sciences during the Scientific Revolution, see Peter Dear, *Discipline and Experience: The Mathematical Way in the Scientific Revolution* (Chicago: University of Chicago Press, 1995); Niccolò Guicciardini, *Reading the Principia: The Debate on Newton's Mathematical Methods for Natural Philosophy from 1687 to 1736* (Cambridge: Cambridge University Press, 1999); Jesseph, *Squaring the Circle*; Michael S. Mahoney, "Mathematization," in Daniel Garber and Michael Ayers (eds.) *The Cambridge History of Seventeenth-Century Philosophy*, vol. 1, 702–756 (Cambridge: Cambridge University Press, 1998); and Paolo Mancosu, *Philosophy of Mathematics and Mathematical Practice in the 17th Century* (New York: Oxford University Press, 1996).

9 The notion that a 'scientific revolution' took place at all has been the subject of much historical debate, and I do not wish to enter on one side or the other. For the present, I am using the term 'Scientific Revolution' as a neutral designation of that period of time—the sixteenth through early eighteenth centuries—of rapid growth in European natural philosophy. For a discussion of the problematic surrounding the traditional conception of revolution, there are a number of excellent sources. Two volumes of essays: David C. Lindberg and Robert S. Westman (eds.) *Reappraisals of the Scientific Revolution* (Cambridge: Cambridge University Press, 1990), and Margaret J. Osler (ed.) *Rethinking the Scientific Revolution* (Cambridge: Cambridge University Press, 2000), have considered this issue from a variety of theoretical and disciplinary perspectives. John A. Schuster's essay "The Scientific Revolution," in R. C. Olby, G. N. Cantor, J. R. R. Christie, and M. J. S. Hodge (eds.) *Companion to the History of Modern Science* (London: Routledge, 1990), pp. 217–242, takes a critical view of the Scientific Revolution as a unified enterprise, as does Stephen Shapin's *The Scientific Revolution* (Chicago: University of Chicago Press, 1998). I. B. Cohen's *Revolution in Science* (Cambridge, MA: Harvard University Press, 1985) examines the historicity of the concept of 'revolution' in the sciences.

There have also been a number of recent works surveying the scientific revolution that explore historiographical themes. Cohen's *The Scientific Revolution: A Historiographic Inquiry* and Shapin's *Scientific Revolution* have already been mentioned; see also John Henry, *The Scientific Revolution and the Origins of Modern Science* (London: Macmillan, 1997), James R. Jacob, *The Scientific Revolution: Aspirations and Achievements, 1500 1700* (Atlantic Highlands, NJ: Humanities Press, 1998), and most recently, Peter Dear, *Revolutionizing the Sciences: European Knowledge and its Ambitions, 1500 1700* (Princeton, NJ: Princeton University Press, 2001), all of which use the term 'revolution' in their titles, despite qualifications about the nature of revolution in science.

10 I am using the term 'constructivism' in the sense given to it by Dear in *Discipline and Experience*, p. 8. I will develop this notion in the following chapters. Briefly, 'constructivism' in mathematics is the belief that mathematical objects are not mind-independent entities or abstractions from physical reality, but rather are artificial 'constructions' produced by the mind that serve as tools in mathematical demonstration. Mathematical constructivism, Dear argues (and I second) was a bridge between discrete individual experiences and the problem of real universals: "a universal concept could be rooted in an empiricist ontology by making a universal claim of constructibility: the universal 'circle' is real because circles can always be drawn." Ibid., p. 219.

11 See Cohen, *The Scientific Revolution*, Chapter 2, particularly pp. 53–96 for a discussion of the importance of the 'great tradition' to the development of the field.

12 Burtt, *Metaphysical Foundations*, p. 104.

13 Ibid., pp. 38, 64.

14 Gary Hatfield, "Metaphysics and the New Science," in David C. Lindberg and Robert S. Westman (eds.) *Reappraisals of the Scientific Revolution* (Cambridge: Cambridge University Press, 1990), pp. 94–95; 117–118.

15 Edward W. Strong, *Procedures and Metaphysics: A Study in the Philosophy of Mathematical-Physical Science in the Sixteenth and Seventeenth Centuries* (Berkeley, CA: University of California Press, 1936), pp. 11–12.

16 Lorraine Daston, "History of Science in an Elegaic Mode: E.A. Burtt's 'Metaphysical Foundations of Modern Physical Science' Revisited," *Isis* 82 (1991), 523–525.

17 Michael S. Mahoney, "The Mathematical Realm of Nature," in Daniel Garber and Michael Ayers (eds.) *The Cambridge History of Seventeenth-Century Philosophy* (Cambridge: Cambridge University Press, 1998), p. 703.

18 Michel Blay, *Reasoning with the Infinite* (Chicago: University of Chicago Press, 1998), pp. 3–5.

19 Jacob Klein, *Greek Mathematical Thought and the Origin of Algebra* (Cambridge, MA: MIT Press, 1968), p. 152.

20 Paolo Mancosu, *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century* (New York: Oxford University Press, 1996), p. 5.

21 See particularly Helena M. Pycior, *Symbols, Impossible Numbers and Geometric Entanglements: British Algebra through the Commentaries on Newton's 'Universal Arithmetick'* (Cambridge: Cambridge University Press, 1997).

22 Ibid., p. 7.

23 See especially Antonio Perez-Ramos, *Francis Bacon's Idea of Science and the Maker's Knowledge Tradition* (Oxford: Clarendon Press, 1988), and Simon Schaffer, "Machine Philosophy: Demonstration Devices in Georgian Mechanics," *Osiris* 9 (1994), 157–182.

24 Dear, *Discipline and Experience*, p. 9.

25 Funkenstein, *Theology*, p. 12.

1 Realists and Nominalists

1 On the translation and dissemination of Aristotelian texts in the Middle Ages, see Edward Grant, *The Foundations of Modern Science in the Middle Ages: Their Religious, Institutional and Intellectual Contexts* (Princeton, NJ: Princeton University Press, 1996), Chapter 2.

2 An excellent general resource is the recent reference volume, Jacques Brunschwig and G. E. R. Lloyd (eds.) *Greek Thought: A Guide to Classical Knowledge*, trans. Catherine Porter (Cambridge, MA: Harvard University Press, 2000).

3 See Anthony A. Long, "Language," in *Greek Thought*, pp. 341–343.

4 This theory clearly recognizes that language has a conventional aspect, in that particular names are chosen by convention to represent particular objects. This is true of every theory of language I will consider in this study, but such a belief does not, in every case, equal what I will refer to as a 'conventionalist' philosophy of language. A truly conventionalist theory finds no correspondence whatever between words and things, because it denies that the *ideas* signified in language, and not merely the words, correspond directly to external reality. The philosophy of language is distinguished from grammar on this basis: while grammatical study is concerned simply with the proper arrangement and deployment of written and spoken words in a particular language (and thus is usually conventional), the philosophy of language investigates how language functions in the mechanism of cognition to produce knowledge (reliable or otherwise) of the external world.

This issue introduces a difficulty in using the term 'language' to refer interchangeably to individual speech acts, particular to a language (such as Latin or English), and to the language faculty in general. The Greeks recognized that these were separate cases, as did most of the authors I will consider in this study, and when I refer to beliefs about 'language,' unless otherwise noted, I am indicating only the latter, and most general, cognitive function. In modern linguistics, this distinction was first and most famously applied by Ferdinand de Saussure, whose structuralist theory separated 'speech' (*parole*), 'a language' (*la langue*), and 'language' in general (*langue*). *Course in General Linguistics*, edited by Charles Bally and Albert Sechehaye, in collaboration with Albert Reidlinger; trans. Wade Baskin (London: Fontana, 1974).

5 Aristotle, *On Interpretation*, I, 16a. All citations for Aristotle are to *The Works of Aristotle*, trans. W. D. Ross (Oxford: Clarendon Press, 1908–52), and will be cited by chapter and line number.

6 Deborah K. W. Mondiak has published a thorough and incisive study of Aristotle's philosophy of language. She argues that Aristotle and Plato agree on a number of vital issues, including a belief that the mind directly apprehends universal categories, and that "the deep structure of language is natural and mirrors reality." Mondiak, *Aristotle's Philosophy of Language and Meaning* (Cambridge: Cambridge University Press, 2001), p. 109.

7 Aristotle, *Posterior Analytics*, I, 10, 76b.

8 Epicurus, *Letter to Herodotus*, 46a–52a; 75b–76a. Trans. Russel M. Geer (New York: Macmillan, 1964).

9 I will discuss the relationship between a physical ontology of atoms and epistemological conventionalism and nominalism in the next chapter.

10 Plato, *Timaeus*, 30a–b; 55a–56b. Trans. Desmond Lee (London: Penguin, 1977).

11 Ibid., 28a–29a.

12 For a more detailed explanation of Plato's philosophy of mathematics, see Robert S. Brumbaugh, *Plato's Mathematical Imagination* (Bloomington, IN: Indiana University Press, 1954), esp. pp. 193–198, 209–229. Also, Anne Freire Ashbaugh's study of Platonic cosmology, *Plato's Theory of Explanation: A Study of the Cosmological Account in the 'Timaeus'* (Albany, NY: State University of New York Press, 1988) contains a significant exposition of Plato's mathematical beliefs.

13 Plato, *Theaetetus*, 185, trans. Benjamin Jowett, in *The Dialogues of Plato* (Oxford: Clarendon Press, 1953).

14 Plato, *Meno*, 76, trans. Benjamin Jowett, in *Dialogues*.

15 Mondiak, *Aristotle's Philosophy*, p. 3.

16 Thomas Heath notes that "the historian of mathematics can glean from his treatises valuable hints as to the contents of the text-books in use immediately before the time of Eulcid." Heath, *Mathematics in Aristotle* (Oxford: Oxford University Press, 1949), p. 1. There have been very few book-length studies of Aristotle's philosophy of mathematics; along with Heath's work, Hippocrates George Apostle's *Aristotle's Philosophy of Mathematics* (Chicago: University of Chicago Press, 1952) is a good, if somewhat dated, resource. As a general survey of Greek mathematics, Jacob Klein's *Greek Mathematical Thought and the Origin of Algebra*, trans. Eva Braun (1934 and 1936; reprint, Cambridge, MA: MIT Press, 1968) is invaluable.

17 Jonathan Barnes cautions, however, that this does not mean Aristotle denied the *existence* of abstract entities. On the contrary, Barnes maintains, while such an abstract concept as 'justice' "has a parasitical existence, not an independent existence . . . this is not to say that justice does not exist at all." "Metaphysics," in Jonathan Barnes (ed.) *The Cambridge Companion to Aristotle* (Cambridge: Cambridge University Press, 1995). Whereas Plato believed an abstract notion like 'beauty' had independent existence in the realm of the Forms, Aristotle held that all such abstract entities existed only insofar as the depended on some other object—'beauty' exists only as a quality of some object that is beautiful.

18 If logic is founded on definitions, and truth is determined by definitions only, then logical truth might in some sense be 'weaker' than Plato's unambiguous idealism.

19 Aristotle, *Posterior Analytics*, II, 7, 92b.

20 Ibid., II, 8, 93a. Mondiak confirms this interpretation, claiming that Aristotle equates "the real definition [with] the expression of an existence of an extramental object." Mondiak, *Aristotle's Philosophy*, p. 266.

21 Aristotle, *Metaphysics*, 1076a.

22 Ibid., 1076b–1077a.

23 Ibid., 1077b.

24 Aristotle discusses the difference between arithmetic and geometry in *Categories*, 4b–5a, and outlines his hierarchy of the mathematical disciplines in *Posterior Analytics*, 87a.

25 Mondiak also supports this linguistic/empirical reading of Aristotle's philosophy of mathematics. She contends mathematical objects, according to Aristotle, are abstract, but they are reached by "stripping away the characteristics of concrete objects." This is similar to the way Aristotle arrives at any universal category—such as 'man'—from particular objects, like 'this man.' Mondiak, *Aristotle's Philosophy*, p. 122.

26 Lucas Siorvanes, *Proclus: Neo-Platonic Philosophy and Science* (Edinburgh: Edinburgh University Press, 1996), pp. 34–37. See also Glenn R. Morrow's translation and introduction to this work, *Proclus: A Commentary on the First Book of Euclid's Elements* (Princeton, NJ: Princeton University Press, 1970).

27 Morrow, "Introduction," *Proclus' Philosophy of Mathematics*, p. xxxiii. This statement may be rather strong, given the extensive analysis of mathematics in Aristotle's writings, but the point is taken: the 'golden era' of classical mathematics—Euclid, Appolonius, Archimedes, etc.—was more concerned with technical exposition than metaphysics, and Proclus provides an invaluable insight into the philosophical basis for the 'new' mathematical methods.

28 Proclus, *Commentary*, I, i, p. 4.

29 According to Proclus, *Commentary*, I, vi, p. 13:

> If, however, mathematical forms do not exist by abstraction from material things or by the assembling of the common characters in particulars, nor are in any way later-born and derivative from sense objects, of necessity the soul must obtain them either from herself or from *Nous*, or from both herself and that higher intelligence.

30 The classic work on the history of skepticism is Richard H. Popkin's *The History of Skepticism from Erasmus to Descartes* (Assen: Van Gorcum, 1960), which has been supplemented by Popkin in several succeeding works that cover the Enlightenment and modern eras.

31 Ibid., p. xii.

32 Pierre Gassendi, *Exercises Against the Aristotelians* in *The Selected Works of Pierre Gassendi*, ed. and trans. Craig B. Brush (New York: Johnson Reprint Co., 1972), p. 97; Gassendi, *Opera Omnia*, 6 vols. (Lyon, 1658; reprint, Stuttgart: Friedrich Frommann Verlag, 1964), vol. III, pp. 202a–202b. I will discuss this link in detail in the next chapter.

33 Sextus, *Outlines*, vol. I, vii, 15, pp. 5–6.

34 Sextus, *Outlines*, vol. II, xx, 156–163, pp. 185–187.

35 Sextus, *Outlines*, vol. II, xx, 167, p. 188.

36 Popkin notes that Pyrrhonian philosophy was "almost unknown" until the mid-1500s. *Skepticism*, p. xiii.

37 It has been a temptation for historians to view the influence of scholasticism on the development of early modern natural philosophy in primarily negative terms: the Scientific Revolution grew out of a disenchantment with Aristotelian logic, physics, and metaphysics, and stands as a reaction 'against' an outmoded and unwieldy philosophical system. It is certainly true that the sixteenth and seventeenth centuries saw their share of polemical attacks against the teaching of the 'schoolmen,' but it is equally true that so-called scholastic methods were preserved in Jesuit and other Catholic traditions of scholarship well into the late seventeenth century, and that many of the older institutional structures that governed educational practices (curriculum, disputations, etc.) remained in place through the height of the Scientific Revolution.

38 See particularly Joseph Owens, "Aristotle and Aquinas," in Norman Kretzmann and Eleonore Stump (eds.) *The Cambridge Companion to Aquinas* (Cambridge: Cambridge University Press, 1993) for a thorough discussion of the relation between Aristotle's and Aquinas' thought.

39 Scott MacDonald, "Theory of Knowledge," in *The Cambridge Companion to Aquinas*, p. 169.

40 Aquinas does not discuss mathematics in much depth, but what he does propose regarding mathematical objects is consistent with his approach to representation in general: mathematical objects exist only as abstracted from quiddity, and mathematics provides formal knowledge rather than knowledge of particulars. C. H. Lohr notes, however, that Aquinas was not particularly receptive towards mathematical astronomy or mathematized physics, preferring the qualitative physics of Aristotle to the theoretical technical astronomy of Ptolemy. This, Lohr argues, was due to a desire to preserve the "absolutely certain, metaphysical principles" of order and harmony in Aristotelian philosophy, against the astronomers' clumsy epicycles and deferents designed to mathematically save the phenomena. C. H. Lohr, "The Medieval Interpretation of Aristotle," in *The Cambridge History of Later Medieval Philosophy*, p. 94. See also Sten Ebbesen, "Ancient Scholastic Logic as the Source of Medieval Scholastic Logic," in *The Cambridge History of Later Medieval Philosophy*, pp. 101–127.

41 Meyrick H. Carre, *Realists and Nominalists* (Oxford: Oxford University Press, 1946), p. 99.

42 Martin M. Tweedale, "Abelard and the Culmination of the Old Logic," in *The Cambridge Companion to Later Medieval Philosophy*, pp. 143–154. Carre calls Abelard's philosophy a "qualified form of Nominalism." Carre, p. 61.

43 Marilyn McCord Adams, "Universals in the Early Fourteenth Century," in *The Cambridge Companion to Later Medieval Philosophy*, pp. 413–415.

44 Carre, *Realists and Nominalists*, pp. 120–122; Serene, "Demonstrative Science," pp. 514–515.

45 As Paul Vincent Spade notes, it was possible to reject universals, as John Burdian did, without dropping Aristotle's set of the ten real ontological categories of existence. Conversely, authors such as Walter Burley were critical of the categories but maintained real universals. While Burdian might be considered a 'nominalist' and Burley a 'realist,' it is apparent that the modern definitions do not strictly apply to the medieval period. I am focusing on Ockham not because I consider him a 'real' nominalist, but rather because his philosophy promotes those views that came to be most closely associated with nominalism in later times. Paul Vincent Spade, "Ockham's Nominalist Metaphysics," in Paul Vincent Spade (ed.) *The Cambridge Companion to Ockham* (Cambridge: Cambridge University Press, 1999), p. 100.

46 Spade, "Ockham's Nominalist Metaphysics," p. 100.

47 William of Ockham, *Summa Logicae*, trans. and ed. Michael J. Loux, *Ockham's Theory of Terms: Part I of the Summa Logicae* (South Bend, IN: St. Augustine's Press, 1998), vol. I, xvii, p. 86.

48 Ockham, *Summa*, vol. I, i, p. 50.

49 Ibid., vol. I, xv, p. 82.

50 Claude Panaccio, "Semantics and Mental Language," in *The Cambridge Companion to Ockham*, p. 61.

51 André Goddu, "Ockham's Philosophy of Nature," in *The Cambridge Companion to Ockham*, pp. 151–152.

52 Ockham, *Quodlibetal Questions*, trans. Alfred J. Freddoso and Francis E. Kelley (New Haven, CT: Yale University Press, 1998), 4th quodlibet, Q 25, pp 343–350.

53 Ockham refers the reader to Anselm, *Monologion*, Chapter 25.

54 Ockham, *Quod. Questions*, p. 345.

55 Ibid., p. 346.

56 Bert Hanson, *Nicole Oresme and the Marvels of Nature: A Study of his* De causis mirabilium *with Critical Edition, Translation, and Commentary* (Toronto: Pontifical Institute of

Mediaeval Studies, 1985), p. 106, summarizes the relation between the theological and ontological implications of nominalism very nicely:

> Of course it is not the mere acknowledgement of the two powers that is characteristic of Nominalists, but rather the employment and the implications of the distinction. For Realists, there are in the world essences or natures of things that are the foundations of the universals we recognize. These essences, by virtue of their real and not merely conceptual existence, present limits to some operations of the power of God. But for Nominalists, by whose understanding the world contains only individuals with no general natures or shared essences, God has the power to handle these individuals in ways that He could not if they had essences.

See also Amos Funkenstein, *Theology and the Scientific Imagination from the Middle Ages to the Seventeenth Century* (Princeton, NJ: Princeton University Press, 1986), Chapter 3.

57 Ockham, *Super quatour libros sententiarum subtilissimae quaestiones earumdemque decisions* I, dist. 13, qu. 1; in Stephen Chak Tournay, *Ockham: Studies and Selections* (La Salle, IL: Open Court, 1938), p. 132.

58 Ockham, *Sentences* I, dist. 2, qu. 7; Tournay, *Ockham*, p. 120.

59 Ockham, *Quod. Quest*. III, qu. 5, pp. 183–184.

60 Ockham, *Quod. Quest.* III, qu. 10, pp. 201–203.

61 Ockham, *Sentences* I, dist. 38, qu. 1, in Philotheys Boehner, *Ockham: Philosophical Writings* (London: Thomas Nelson, 1957), pp. 133–134.

62 Ockham, *Summa*, I, xvii, p. 86. Goddu writes, "however deviant he may appear, Ockham remained an Aristotelian or as Aristotelian as he could." Goddu, "Ockham's Philosophy of Nature," p. 166.

63 Nicholas of Autrecourt, *The Universal Treatise*, trans. L. A. Kennedy, R. E. Arnold, and A. E. Millward (Milwaukee, WI: Marquette University Press, 1971), p. 68 [204]. The number in square brackets refers to the pagination in the original Latin manuscript.

64 Autrecourt, *Universal Treatise*, p. 35 [183].

65 Ibid., p. 36 [184].

66 Ibid., p. 105 [228].

67 See Serene, "Demonstrative Science," p. 515 for a short but incisive discussion of Ockham and Autrecourt's differing views regarding demonstrative knowledge.

68 Goddu, "Ockham's Philosophy of Nature," p. 149.

69 Autrecourt, *Universal Treatise*, p. 72 [207].

70 E. J. Ashworth, "The Eclipse of Medieval Logic," in *The Cambridge History of Later Medieval Philosophy*, p. 787.

71 G. A. Padley, *Grammatical Theory in Western Europe: 1500 1700* (Cambridge: Cambridge University Press, 1976), pp. 16–29, presents a clear and insightful chronological narrative of Renaissance grammatical trends.

72 Timothy J. Reiss, *Knowledge, Discovery and Imagination in Early Modern Europe: The Rise of Aesthetic Rationalism* (Cambridge: Cambridge University Press, 1997), p. 2.

73 Lisa Jardine, "Humanism and the Teaching of Logic," in *The Cambridge History of Later Medieval Philosophy*, p. 797.

74 All these English translations are available as modern reprints: *Latin Grammar* and *Rudiments of Latin Grammar* are part of the *English Linguistics* facsimile reprint series published by the Scolar Press (nos. 305 and 306 respectively), ed. R. C. Alston (Menston: The Scolar Press, 1971), while *The Logicke of the Most Excellent Philosopher P. Ramus Martyr*, trans. Roland MacIlmaine (1574) appears, ed. Catherine M. Dunn (Northridge, CA: San Fernando Valley State College, 1969). The 'martyr' in the title of the latter work refers to Ramus's murder by the mob in the St. Bartholomew's Day Massacre.

75 Padley, *Grammatical Theory*, p. 111; Walter J. Ong, *Ramus, Method, and the Decay of Dialogue: From the Art of Discourse to the Art of Reason* (Cambridge, MA: Harvard University Press, 1958), p. 247. Padley argues that the 'formalist' approach was "doomed to

failure," a contention I reject. Ong's work, though dated, is still the most comprehensive and penetrating study of Ramus's life, work, and intellectual milieu.

76 Petrus Ramus, *Logicke*, I, xxiii, pp. 29–30. I have modernized the spelling in MacIllmaine's 1574 English translation.

77 Reiss argues that according to Ramus, logic "did allow one to get at meaningful truths," but that these truths "were truths of relation and practical truths of manipulation." Reiss, *Knowledge, Discovery and Imagination*, p. 121.

78 Nicholas Jardine, *The Birth of History and Philosophy of Science: Kepler's A Defence of Tycho against Ursus with Essays on its Provenance and Significance* (Cambridge: Cambridge University Press, 1988), p. 234. Jardine goes on to note that Ramus denied the traditional Aristotelian division between pure and 'mixed' mathematics; for Ramus, all mathematics should be applied towards "solving specific mathematical problems." Jardine calls Ramus's approach "fictionalist or instrumentalist," and notes that in modern philosophical terms, this kind of 'instrumentalism' is a close relative of 'constructivism' in mathematics. I believe this correspondence suggests there is some justification for aligning Ramus with the later constructivist tradition in seventeenth-century mathematics. Jardine, p. 236.

79 Reiss cites the importance of a Latin grammar by 'Francisco Sanchez,' (*Knowledge, Discovery and Imagination*, p. 27), but he has apparently confused Francisco Sanches with Juan Sanchez, the real author of the 1586 work, *Principos de la grammatica latina*.

80 Francisco Sanches, *That Nothing is Known: (QVOD NIHIL SCITUR)*, trans. Douglas F. S. Thomson (Cambridge: Cambridge University Press, 1988).

81 Ibid., pp. 172, 200.

82 Ibid., p. 168.

83 Ibid., p. 174.

84 Ibid., p. 219.

85 Ibid., p. 213.

86 Ibid., p. 236.

87 Ibid., p. 244.

2 Ontology Recapitulates Epistemology

1 On the revival of atomism in the seventeenth century, and in particular Gassendi's role in its promotion, see O. R. Bloch, *La Philosophie de Gassendi: Nominalisme, Matérialisme et Métaphysique* (La Haye: Nijhoff, 1971), Lynn Sumida Joy, *Gassendi the Atomist: Advocate of History in an Age of Science* (Cambridge: Cambridge University Press, 1987), Hugh Kargon, *Atomism in England from Hariot to Newton* (Oxford: Clarendon Press, 1966), and Margaret J. Osler, "Baptizing Epicurean Atomism: Pierre Gassendi on the Immortality of the Soul," in *Religion, Science, and Worldview: Essays in Honor of Richard Westfall* (Cambridge: Cambridge University Press, 1985), pp. 163–183, and Osler, *Divine Will and the Mechanical Philosophy: Gassendi and Descartes on Contingency and Necessity in the Created World* (Cambridge: Cambridge University Press, 1994).

2 For a discussion of the connections between Gassendi and Boyle, see Margaret Osler, "The Intellectual Sources of Robert Boyle's Philosophy of Nature: Gassendi's Voluntarism and Boyle's Physico-Theological Project," in Richard Kroll, Richard Ashcraft, and Perez Zagorin (eds.) *Philosophy, Science and Religion in England, 1640 1700* (Cambridge: Cambridge University Press, 1992), pp. 178–198. On the subject of Newton's exposure to Gassendist physics, see Richard Westfall, "The Foundations of Newton's Philosophy of Nature," *British Journal of the History of Science*, 1 (1962), 171–182, J. E. McGuire and Martin Tamny, *Certain Philosophical Questions: Newton's Trinity Notebook* (Cambridge: Cambridge University Press, 1983).

3 According to Gassendi, "atoms are mobile and active from the power of moving and acting which God instilled in them at their very creation, and which functions with his assent." Pierre Gassendi, *The Selected Works of Pierre Gassendi*, ed. and trans. Craig B.

Brush (New York: Johnson Reprint Co., 1972), p. 399; Gassendi, *Opera Omnia*, 6 vols. (Lyon, 1658; reprint, Stuttgart: Friedrich Frommann Verlag, 1964), vol. I, p. 280a. Subsequent quotations from Gassendi will list translation first (if available), followed by citation from the reprint of the 1658 *Opera*.

4 Several studies of Gassendi have discussed his mathematical interests, but none have seriously considered him a 'philosopher of mathematics.' Alexandre Koyré, speaking at the tercentennial meeting to *celebrate* Gassendi, was frankly dismissive:

> To speak of Gassendi's relation to the science of his time might, at first sight, appear to attempt the impossible. And an injustice. Indeed Gassendi was not a great scientist, and in the history of science, in the strict sense of the term, the place to which he returns is not of great importance.

Alexandre Koyré, *Actes du Congrès du Tricentenaire de Pierre Gassendi (1655 1955)* (Digne: R. Vial, 1957), p. 175. More recent scholarship has, however, taken his mathematical philosophy somewhat more seriously. See Joy, *Gassendi the Atomist*, Barry Brundell, *Pierre Gassendi: From Aristotelianism to a New Natural Philosophy* (Dordrecht: D. Reidel, 1987), and Saul Fisher, *Pierre Gassendi's Philosophy and Science: Atomism for Empiricists* (Leiden: Brill, 2005). Osler does, however, convincingly link Gassendi's nominalism to his rejection of Descartes' realist philosophy of mathematics, but here she focuses more on Gassendi's voluntarist theology than on his nominalist ontology. See Margaret J. Osler, "Divine Will and Mathematical Truth: Gassendi and Descartes on the Status of the Eternal Truths," in Roger Ariew and Marjorie Grene (eds.) *Descartes and His Contemporaries: Meditations, Objections, and Replies* (Chicago: University of Chicago Press, 1995), pp. 145–158.

5 In his recent book, Fisher takes on the laudable task of updating many of the earlier historiographical inadequacies of Gassendi scholarship. Fisher's portrait of Gassendi "depict[s] his philosophical and scientific pursuits as part of one and the same project"—an attempt to construct a consistent physicalist and reliabilist system of empiricist natural philosophy based on Epicurus' physics of atoms. Fisher, *Pierre Gassendi's Philosophy and Science*, p. xxi.

6 Ockham, as well as his later follower Nicholas of Autrecourt, laid the groundwork for Gassendi's nominalist critique of epistemology, and Autrecourt even extended his discussion to a defense of atomism. Osler notes that Gassendi cites the prominent nominalists Gregory of Rimini and Pierre d'Ailly in his *Exercitationes.* See Osler, *Divine Will*, p. 113.

7 It is also fair to say that not every proponent of the mechanical philosophy clearly identified the distinctions between Cartesian and atomistic matter theories: Walter Charleton was an important conduit for Gassendi's philosophy in England, cites Descartes along with Democritus and Epicurus as a subscriber to the atomistic theory. Walter Charleton, *Physiologia Epicuro-Gassendo-Charletoniana: or a Fabrick of Science Natural, upon the Hypothesis of Atoms, Founded by Epicurus, Repaired by Petrus Gassendus, Augmented by Walter Charleton* (London, 1654; reprint, New York: Johnson Reprint Corp., 1966), p. 85. This issue is also noted in Thomas M. Lennon, *The Battle of the Gods and Giants: The Legacies of Descartes and Gassendi, 1655 1715* (Princeton, NJ: Princeton University Press, 1993), p. 98.

8 A number of important works on Gassendi have appeared in the past few decades, but the first truly in-depth modern historical analysis of Gassendi's thought was produced by O. R. Bloch, *La Philosophie de Gassendi: Nominalisme, Matérialisme et Métaphysique*. More recently, attention has been focused on Gassendi by historians of philosophy and of science, notably in Lennon's, Joy's, Brundell's, and Fisher's work. Gassendi has still, however, not been given sufficient attention in the history of science, although historians of philosophy have been more attentive. In 1995, an issue of *Perspectives on Science* (vol. 3) was devoted to consideration of recent scholarship of the Gassendi–Descartes split.

See also the Ariew-Grene collection of essays devoted to the responses of Descartes' *Meditations*, which includes two important papers on Gassendi.

9 M. Glouberman rebuts Lennon's argument in *Gods and Giants* by asserting that "the central development of modernity in its infancy, if modernity is ground to a philosophical prescription, is Cartesianism." Glouberman, "Gods, Giants, Fractals, and the Geometry of Early Modernity: Descartes, Gassendi and the Rise of Science," *Perspectives on Science*, 3 (1995), 481. Glouberman goes on to qualify his argument, stating this is due to Gassendi's and Descartes' views occupying "different levels" of epistemological inquiry, and that Gassendi resided in the older world of "post-Scholastic humanism." Be that as it may, Daniel Garber's introduction to the issue describes Glauberman's attitude more bluntly: "the appearance of Cartesian dominance is the reality." Daniel Garber, "Apples, Oranges, and the Role of Gassendi's Atomism in Seventeenth-Century Science," *Perspectives on Science*, 3 (1995), 425.

10 Joy, *Gassendi the Atomist*, p. 203.

11 Gassendi published his observations of the transits of Mercury and Venus in *Mercurious in sole visus & Venus invisia, Parisiis, Anno 1631, pro vito & admonitione Kepleri*, reproduced in Gassendi, *Opera*, vol. IV, pp. 499–511. Indeed, much of this volume of the *Opera* is devoted to technical astronomy.

12 Gassendi, Letter to du Faur de Pibrac, April 10, 1621, in Gassendi, *Selected Works*, pp. 5–8. He states that he "would note down more carefully the particular things I observed if somebody in your [de Pibrac's] area would give these studies professional attention," indicating that he considers his own attentions 'professional' (as opposed to de Pibrac's amateur curiosity).

13 Gassendi, *Selected Works*, p. 120; *Opera*, vol. IV, p. 478:

> On the Way, I enumerated both my own observations and those which Galileo compiled in support of the theorem that 'If the body we are on is in motion, everything we do and all the things we move will actually take place, and appear to take place, as if it were at rest.' . . . Afterwards, so they would be convinced beyond a doubt, they were taken down to the sea where they were to observe a ship moving with great speed, as well as one at rest, to see whether a stone thrown into the air along the length of the mast . . . would always keep the same distance from the mast.

14 For detailed biographical information about Gassendi's early career, see Jones, *Pierre Gassendi*, pp. 11ff.

15 Gassendi, *Selected Works*, p. 19; *Opera*, vol. III, p. 100:

> [W]hen I was subsequently burdened with teaching philosophy, particularly Aristotle's, for six full years at the Academy of Aix, I always made it a point that my auditors should be able to defend Aristotle well; but as a kind of appendix to the course I also expounded those opinions which would totally undercut Aristotelian dogmas.

16 Pierre Gassendi, *Animadversiones in decimum librum Diogenis Laetrii* (Lyon, 1649). Much of the secondary literature about Gassendi concerns his 'baptism' of Epicurean atomism to defend against the charge of hedonism and atheism commonly leveled against Epicurus. See Osler, "Baptizing," and *Divine Will*.

17 "And so I will conclude here with two points as the capstones which will add a certain confirmation of the statement *quod nihil sciatur*" Gassendi, *Selected Works*, p. 97; *Opera*, vol. III, p. 202. Osler, *Divine Will*, p. 113, also notes that Gassendi cites the prominent nominalists Gregory of Rimini and Pierre d'Ailly in his *Exercitationes*. In the Preface to his first work, *Exercitationes Paradoxicae adversus Aristoteleos*, Gassendi writes, "I remember clearly that when I was indoctrinated in Peripatetic philosophy as a young man, it did not appeal to me in the slightest." Gassendi, *Selected Works*, p. 18; *Opera*, vol. III, p. 99.

18 Gassendi, *Exercises*, p. 19; *Opera* vol. III, p. 99.

19 Howard Jones states that Gassendi probably was introduced to atomism in the late 1620s during a trip to Holland, where he met Isaac Beekmann and J. B. Van Helmont. Jones, *Pierre Gassendi*, p. 282.

20 The most probable reason for this delay is that Gassendi was dissatisfied with his efforts to present the physics as a consistent and clearly expressed system; he was unhappy with the format of the *Animadversiones* and continued to tinker with that work until his death in 1655. See Joy, *Gassendi the Atomist*, p. 71.

21 This is especially evident in Richard Westfall's *The Construction of Modern Science* (Cambridge: Cambridge University Press, 1977): "In a word, Gassendi was the original scissors and paste man, and his book contains all the inconsistencies of eclectic compilations." Ibid., p. 39.

22 Most versions of seventeenth-century corpuscularianism arose from a common goal: to find an alternative to Aristotle's qualitative physics and natural motions. Aristotle's scheme of four terrestrial elements, derived from Empedocles, was an attempt to address the issue of the ultimate constituents of matter using qualitative terms that described bodies according to their essential properties. While Aristotle postulated a 'prime matter' that composed the elements, that matter was a quality-less substratum; the elements were imbued with their qualities as essential components of their natures. As natural philosophers in the seventeenth century made causal explanation a greater priority in physics, they sought to rid physics of what they perceived as 'occult qualities,' or explanations based on recourse to hidden forms or arbitrary essential properties. In doing so they turned to the notion of a universal substance, without qualities, and attempted to imagine schemes by which qualities in larger bodies could be accounted for by the arrangement and motion of basic matter. This, at its root, was the mechanical philosophy.

23 Jones, *Pierre Gassendi*, p. 253.

24 Gassendi, *Selected Works*, p. 388; *Opera* vol. I, p. 183a.

25 Gassendi, *Selected Works*, p. 388; *Opera* vol. I, p. 183a.

26 Gassendi, *Selected Works*, p. 389; *Opera* vol. I, p. 183b.

27 Jones, *Pierre Gassendi*, p. 285; Lucretius, *De rerum natura*, vol. I, 146–483.

28 Jones, *Pierre Gassendi*, p. 288.

29 Gassendi, *Selected Works*, p. 399; *Opera* vol. I, p. 280a.

30 The issue of Gassendi's 'voluntarism' vs. Descartes' deism is best explored in Osler's *Divine Will*, and will not be discussed in detail here.

31 Gassendi, *Selected Works*, p. 327; *Opera* vol. I, p. 80a.

32 This concession dismisses the more radical 'Academic' school of skepticism, which competed with the Pyhrronian version in late antiquity. See Richard H. Popkin's *The History of Skepticism from Erasmus to Descartes* (Assen: Van Gorcum, 1960), which has been supplemented by Popkin in several succeeding works that cover the Enlightenment and modern eras, p. xii.

33 Gassendi, *Selected Works*, p. 327; *Opera* vol. I, p. 80a

34 Gassendi, *Selected Works*, pp. 289–90; *Opera* vol. I, p. 68b

35 Charleton, *Physiologia*, p. 341.

36 Ibid., pp. 341–342.

37 Gassendi, *Selected Works*, pp. 329–30; *Opera*, vol. I, p. 80b.

38 Gassendi, *Selected Works*, p. 330; *Opera*, vol. I, p. 80b.

39 Gassendi, *Selected Works*, p. 333; *Opera* vol. I, p. 81b.

40 For examples of Dee's and Croll's approach to hidden significations, see John Dee, *Monas Hieroglyphica* (1564), trans. C. H. Josten, *Ambix* 12 (1964) pp. 84–221.

41 Gassendi, *Selected Works*, p. 334; *Opera* vol. I, p. 81b.

42 Gassendi, *Selected Works*, p. 335; *Opera* vol. I, p. 82a.

43 Gassendi, *Selected Works*, p. 333; *Opera* vol. I, p. 81b.

44 This is what moves Fisher to classify Gassendi's philosophy as 'physicalist.' Fisher, *Pierre Gassendi's Philosophy and Science*, Chapter 1.

45 Gassendi, *Selected Works*, p. 20; *Opera*, vol. III, p. 100.

46 Gassendi, *Selected Works*, p. 20; *Opera*, vol. III, p. 100.

47 To take an example, the word 'horse,' in scholastic logic, refers to a definition of certain essential qualities that separates horses in nature from all other animals. One may see a donkey and think it is a horse, but that is an error in perception, not in reasoning. In the reverse case, once might use a different word to describe horses, but as long as the 'mental picture' that is being referred to fits the definition of 'horse,' one can be certain that one is speaking of a certain universal category in nature.

48 Gassendi, *Selected Works*, p. 145; *Opera*, vol. III, p.160b.

49 Gassendi, *Selected Works*, p. 43; *Opera*, vol. III, p.159a. He adds "you say, do you agree with that mad opinion of the nominalists . . . ? Right, I do; but I believe that I agree with an opinion that is utterly sane."

50 Gassendi, *Selected Works*, p. 96; *Opera* vol. III, p. 202a.

51 Gassendi quotes Cicero on 'artificial logic' only to show that it is "useless." Gassendi, *Selected Works*, p. 33; *Opera* vol. III, p. 150a.

52 Gassendi, *Selected* Works, p. 97; *Opera* vol. III, p. 202b. This, of course, is the motto of the Pyrrhonists, and also of Francisco Sanches, whose work *Quod nihil sciatur* was familiar to Gassendi.

53 Pierre Gassendi, *Institutio Logica*, reprinted and trans. Howard Jones (Assen: Van Gorcum, 1981) p. 80; *Opera*, vol. I, p. 91a. Gassendi mentions Ramus in the Preface to the *Exercitationes* while discussing his formative influences: "My strength grew greater as I read Ramus and Mirandola. I mention them because I have always made a point of naming those from whom I have profited." Gassendi, *Selected Works*, p. 19; *Opera*, vol. III, p. 99.

54 Gassendi, *Institutio Logica*, p. 84; *Opera*, vol. I, p. 92a.

55 Gassendi, *Institutio Logica*, pp. 84–85; *Opera*, vol. I, p. 92b.

56 Gassendi, *Institutio Logica*, pp. 85–87; *Opera*, vol. I, pp. 93a–b.

57 Gassendi, *Selected Works*, p. 85; *Opera* vol. I, p. 92b.

58 Gassendi, *Selected Works*, p. 97; *Opera* vol. III, p. 202b.

59 Gassendi, *Selected Works*, p. 107; *Opera* vol. III, p. 209a.

60 Gassendi was certainly aware of the Renaissance debate over the certainty of mathematics involving Clavius, Piccolomini, Barozzi, Pereyra, and others. He quotes, in the *Exercitationes*, from Pereyra's 1576 *De communibus omnium rerum naturalium principiis et addectionibus libri quindecim*, and as Mancosu notes, "passages from Gassendi and Barrow on the nature of mathematics are incomprehensible without referring back to the *Quaestio*." Paolo Mancosu, *Philosophy of Mathematics and Mathematical Practice in the 17th Century* (New York: Oxford University Press, 1996), p. 92.

61 Gassendi, *Selected Works*, p. 107; *Opera* vol. III, p. 209a.

62 Gassendi, *Selected Works*, p. 107; *Opera* vol. III, p. 209a.

63 Gassendi, *Selected Works*, p. 108; *Opera* vol. III, p. 209a.

64 Galileo's mathematics has, now somewhat infamously, been classified as 'Platonic' by Alexandre Koyré, *Galileo Studies*, trans. John Mepham (Atlantic Highlands, NJ: Humanities Press Inc., 1978). However, this characterization has been questioned by more recent scholarship, including Peter Machamer, "Galileo's Machines, his Mechanics, and his Experiments," in Peter K. Machamer (ed.) *The Cambridge Companion to Galileo* (Cambridge: Cambridge University Press, 1998), and Rivka Feldhay, "The Use and Abuse of Mathematical Entities: Galileo and the Jesuits Revisited," in *The Cambridge Companion to Galileo*. Their reinterpretations do not, however, deny that Galileo believed mathematical entities were fundamentally real.

65 Joy, *Gassendi the Atomist*, pp. 91–92. As Joy notes, Gassendi incorporated a "strong bias against introducing mathematical arguments into his answers and his doubts about the usefulness of determining the properties of strictly mathematical points were not shared by the majority of Poysson's nine other correspondents."

66 Ibid., p. 102. She also states that Gassendi believed a mathematician "identifies physical points and magnitudes and attributes the appropriate names to these individuals

whenever he can show that their physical properties adequately resemble the properties specified in the definitions of their names."

67 Gassendi to Mersenne, December 13, 1635, translated in Joy, *Gassendi the Atomist*, p. 103.

68 René Descartes, *The Philosophical Writings of Descartes*, 3 vols., trans. J.. Cottingham, R. Stoothoff and D. Murdoch (Cambridge: Cambridge University Press, 1984), vol. 2, p. 269.

69 Descartes, *Philosophical Writings*, vol. 2, p. 269.

70 Descartes, *Philosophical Writings*, vol. 2, p. 44.

71 Descartes, *Philosophical Writings*, vol. 1, p. 231.

72 See particularly Stephen Gaukroger, "The Nature of Abstract Reasoning: Philosophical Aspects of Descartes' Work in Algebra," in John Cottingham (ed.) *The Cambridge Companion to Descartes* (Cambridge: Cambridge University Press, 1992), pp. 102ff. He states (ibid., p. 104):

> In this respect, Descartes inaugurates a development in which the range of items coming under the category of 'number' is expanded and consolidated as the generality of algebra is increased and its rules of operation define new kinds of entity as numbers.

73 Jacob Klein, *Greek Mathematical Thought and the Origin of Algebra* (Cambridge, MA: MIT Press, 1968), p. 205.

74 Klein, *Greek Mathematical Thought*, p. 210.

75 Descartes to Mersenne, April 15, 1630, in Paul Tannery (ed.) *Correspondence du P. Marin Mersenne* (Paris: Presses Universitaires de France, 1945), vol. 2, p. 431.

76 Descartes, *Philosophical Writings*, vol. 1, p. 247. A great difficulty in presenting Descartes' philosophy in a limited space is owed to the fact that his ideas, from the *Meditations* through the *Principles*, changed over time, and that in later works he frequently contradicted positions he had explicitly maintained in earlier ones. For the purposes of the present study, I am not attempting to definitively characterize Descartes' views throughout his lifetime, although I maintain that my characterization is accurate for the period roughly covering 1630–1644, especially insofar as it relates to his views about mathematics and representation as epistemological questions. Of vital importance is properly presenting these views as they were received by his contemporaries—most particularly by Gassendi.

77 Descartes, *Philosophical Writings*, vol. 2, pp. 274–275.

78 Ibid., vol. 2, p. 275.

79 Gassendi, *Disquisitio Metaphysica*, in *Selected Works*, pp. 165–166; *Opera*, vol. III, p. 279b.

80 Gassendi, *Selected Works*, p. 253; *Opera* vol. III, p.377b.

81 Gassendi, in Descartes, *Philosophical Writings*, vol. 2, p. 229. Emphasis in original.

82 Gassendi, in Descartes, *Philosophical Writings*, vol. 2, p. 228.

83 Gassendi, in Descartes, *Philosophical Writings*, vol. 2, p. 241.

84 Descartes, *Philosophical Writings*, vol. 2, p. 262.

85 Lennon, *The Battle of the Gods and Giants*, p. 215.

86 Gassendi, *Selected Works*, p. 257; *Opera*, vol. III, p. 378b.

87 Gassendi's immediate group of friends and admirers in the 1640s and 1650s was a group of Parisian intellectuals sometimes described as '*libertins*.' This group included Pierre Petit, Samuel Sorbière, François Bernier, Gilles de Launay and Marin Cureau de La Chambre, who met in various configurations for several decades in the homes of wealthy patrons. There were several such groups in Paris at the time, and they formed the prototypes for not only the French Royal Academy, but also for the salon culture of the eighteenth-century *philosophes*. Gassendi also had an extensive network of friends outside of his immediate circle and was an intimate of Mersenne and Hobbes.

3 British Empiricism, Nominalism, and Constructivism

1 For recent studies of the British Royal society, see Michael Hunter, *The Royal Society and its Fellows, 1660 1700: The Morphology of an Early Scientific Institution*, 2nd edn (London: British Society for History of Science, 1994); "The Early Royal Society and the Shape of Knowledge," in Donald R. Kelley and Richard H. Popkin (eds.) *Shapes of Knowledge from the Renaissance to the Enlightenment* (Dordrecht: Kluwer Academic, 1991), pp. 189–202; and with Paul B. Wood, "Towards Solomon's House: Rival Strategies for Reforming the Early Royal Society," *History of Science* 24 (1986), 49–108. On the subject of the formation of scientific societies in France, the classic work is still Harcourt Brown's *Scientific Organisations in Seventeenth Century France (1620 1680)* (Baltimore, MD: 1934; reprinted, New York: Russell & Russell, 1967). On scientific societies in France, see Alice Stroup, *A Company of Scientists: Botany, Patronage and Community at the Seventeenth-Century Parisian Royal Academy of Sciences* (Berkeley, CA: University of California Press, 1990), and David J. Sturdy, *Science and Social States: The Members of the Académie des Sciences 1666 1750* (Rochester, VA: Boydell Press, 1995). Finally, useful general surveys of scientific community and culture include Lisa Jardine, *Ingenious Pursuits* (London: Little, Brown, 1999), and James E. McClellan, III, *Science Reorganized: Scientific Societies in the Eighteenth Century* (New York: Columbia University Press, 1985).

2 G. A. Padley, *Grammatical Theory in Western Europe: 1500 1700* (Cambridge: Cambridge University Press, 1976), pp. 141–142.

3 Douglas M. Jesseph, *Squaring the Circle: The War between Hobbes and Wallis* (Chicago: University of Chicago Press, 1999), pp. 208–209.

4 Amos Funkenstein, *Theology and the Scientific Imagination from the Middle Ages to the Seventeenth Century* (Princeton, NJ: Princeton University Press, 1986), p. 81.

5 Noel Malcolm, "A Summary Biography of Hobbes," in Thomas Sorell (ed.) *The Cambridge Companion to Hobbes* (Cambridge: Cambridge University Press, 1996) pp. 21–24. On the history of the Newcastle Circle, see Robert H. Kargon, *Atomism in England from Hariot to Newton* (Oxford: Clarendon Press, 1966), especially Chapters 1 and 2.

6 In a letter two years later sent directly to Gassendi, Hobbes commented that "according to my judgement of men, your knowledge is greater than all mortals', and your virtue is even greater than your knowledge." Hobbes to Gassendi, 22 September 1649, in Thomas Hobbes, *The Clarendon Edition of the Works of Thomas Hobbes: The Correspondence*, 2 vols., ed. Noel Malcolm (Oxford: Clarendon Press, 1994), vol. 1, p. 179. As Gassendi's health declined in the early 1650s, Hobbes's inquiries after Gassendi were solicitous and respectful; indeed, they often have the tone of a pupil addressing a beloved teacher, despite the fact that Hobbes was four years older than Gassendi. It also appears from the tone of several letters that Hobbes was somewhat anxious for reassurance that the respect was mutual, as the following letter seems to indicate: Hobbes to Gassendi, 10 July, 1654, *Correspondence*, vol. 1, p. 185:

> I was very anxious about your illness, which I thought was a disease of the lungs, and a very dangerous one. But now I feel no anxiety—except that which I feel when I consider the mortality of all men. I am glad that God did not decide to rob the world of such a treasure, and I reckon on at least another twenty years of your life. M. du Verdus also told me in his letter (in terms more laudatory than I deserve) that you continue to hold me in the same high esteem.

7 Noel Malcolm, "Biographical Register," in Hobbes, *Correspondence*, vol. 2, p. 836; Pierre Gassendi, *Opera Omnia.* 6 vols. (Lyon, 1658, reprint, Stuttgart: Friedrich Frommann Verlag, 1964), vol. I, sig. 2.

8 Sorbière tells Hobbes that in the wake of Gassendi's failure to produce the long-anticipated summary of his philosophical system (the *Syntagma*, which did not appear until after his death) those who had been looking to both Hobbes and Gassendi "have their

eyes on you now that that mountain has brought forth the ridiculous mouse and made fools of us." It appears from the context that what had been anticipated of both was a response to the Cartesian system, since the same letter goes on to criticize another colleague, Henricus Regius, for being "too much 'enslaved to the hypothesis,' namely that the sharp mind of Descartes has penetrated that innermost sanctuary of nature, which perhaps no man will ever fully inspect or know with certainty." Sorbière to Hobbes, 21 May, 1646, in Hobbes, *Correspondence*, vol. 1, p. 129. Sorbière was particularly interested to have Hobbes's new works on optics and physics, and elsewhere urges Hobbes to prod Gassendi to produce his own physical system; evidently, he believes that they will address the same issues. As evidence of Hobbes's interest in Gassendi's work, Du Verdus, writes to Hobbes to apologize for failing to deliver "your copy of M. Gassendi's *Epicurus* [the *Animadversiones*] for more than four months," since Hobbes "may perhaps have been in a hurry to read these books in order to judge and refute some of the theories they contain which are so ill-founded." Du Verdus to Hobbes, 20 August, 1654, Hobbes, *Correspondence*, vol. 1, p. 195.

9 The issue of accuracy in translations of *De corpore* has been raised by several modern scholars and will be discussed in greater detail below. The standard translation is found in *The English Works of Thomas Hobbes of Malmsbury; Now First Collected and Edited by Sir William Molesworth, Bart.*, 11 vols. (London: John Bohn, 1839–45), vol. 1. Since, however, this work has appeared with such varied titles as '*De corpore*,' '*Elements of Philosophy, the First Section concerning Body*,' and '*Elementorum philosophiae sectio prima*,' I will for simplicity's sake refer to it as '*De corpore*' in the body of my text. I will, however, specify the edition and translation in citations.

10 "If the disease had not intervened I should, I think, have completed the first part of my 'Philosophy,' which is about body; but as things now are, you can expect to receive that part at about Whitsun." Hobbes to Sorbière, 27 November, 1647, in Hobbes, *Correspondence*, vol. 1, p. 164.

11 Thomas Hobbes, *Leviathan*, ed. Richard Tuck (London, 1651; reprint, Cambridge: Cambridge University Press, 1996), 1.1, p. 13.

12 Ibid., Book. 1, Chapter. 2, pp. 15–16.

13 Ibid., Book. 1, Chapter 2, p. 19.

14 Ibid., Book. 1, Chapter 3, p. 20.

15 Ibid., Book. 1, Chapter 3, p. 21.

16 Thomas Hobbes, *Computato Sive Logica: Logic*, trans. Aloysius Martinich (London, 1655; reprint, New York: Abaris Books, 1981), Chapter 2, §1, p. 195.

17 Ibid., Chapter 2, §2, pp. 195–197.

18 Ibid., Chapter 2, §2, p. 197.

19 Ibid., Chapter 2, §3, p. 197.

20 Ibid., Chapter 2, §3, p. 197.

21 Hobbes, *Leviathan*, Book. 1, Chapter 4, p. 26.

22 Hobbes, *Computato Sive Logica*, Chapter 2, §9, p. 207.

23 Ibid., Chapter 2, §4, p. 199; Chapter 2 §5, p. 201.

24 Hobbes, *Leviathan*, Book. 1, Chapter 4, p. 27:

> Thus the consequence found in one particular, comes to be registered and remembered, as an Universall rule; and discharges our mentall reckoning, of time and place; and delivers us from all labour of the mind, saving the first; and makes that which was found true *here*, and *now*, to be true in *all times* and *places*.

25 Hobbes, *Computato Sive Logica*, Chapter 2, §10, p. 209. According to Padley, Hobbes was "the exemplar of this tendency to extreme Nominalism" which has its roots in the skeptical philosophy of the late scholastic period.

26 Hobbes, *Leviathan*, Book. 1, Chapter 4, pp. 27–28.

27 Hobbes, *Computato Sive Logica*, Chapter 5, §1, p. 271.

28 Ibid., Chapter 3, §7, p. 235.
29 Jesseph, *Squaring the Circle*, p. 199.
30 Hobbes's set of objections were published as the 'Third Set,' and are taken from *The Philosophical Writings of Descartes*, 3 vols., trans. J. Cottingham, R. Stoothoff and D. Murdoch (Cambridge: Cambridge University Press, 1984).
31 Thomas Hobbes, "Third Set of Objections with the Author's Replies," in *The Philosophical Writings of Descartes*, vol. 2, p. 126.
32 Hobbes, "Objections," p. 125.
33 Ibid., p. 136. Hobbes's nominalism guides this response to Descartes, since Descartes has stated that "there is still a determinate nature, or essence, or form" of objects held in the mind (the example at hand is a triangle) "which is immutable and eternal, and not invented or dependent on my mind." Descartes, *Philosophical Writings*, vol. 2, pp. 44–45.
34 Hobbes, *Computato Sive Logica*, Chapter 6, §1, p. 289. See especially Douglas Jesseph, "Hobbes and the Method of Natural Science," in *The Cambridge Companion to Hobbes*, pp. 86–107. Hobbes, *Elements of Philosophy: The First Part, Concerning Body*, in *The English Works of Thomas Hobbes*, vol. 1, Book. 3, Chapter 20, §6, p. 312.
35 Hobbes, *Leviathan*, Book. 1, Chapter 4, p. 29.
36 Hobbes, *Computatio Sive Logica*, Chapter 1, §2, p. 177.
37 Hobbes, *Leviathan*, Book. 1, Chapter 5, p. 32.
38 Hobbes, *English Works*, vol. 7, p. 307.
39 Hobbes, *Computato Sive Logica*, p. 301.
40 Helena M. Pycior, *Symbols, Impossible Numbers and Geometric Entanglements: British Algebra through the Commentaries on Newton's 'Universal Arithmetick,'* (Cambridge: Cambridge University Press, 1997), p. 140.
41 Hobbes, *English Works*, vol. 1, p. 140.
42 Hobbes, *Elements*, Book 3, Chapter 15, §2, p. 207.
43 Jesseph, *Squaring the Circle*, p. 103.
44 Hobbes, *Elements*, Book 3, Chapter 20, §6, pp. 316–317.
45 Pycior helps to clarify this last point: "In invention, symbols provided a shorthand to record the mathematician's progress. Symbolical formulation, however, prolonged rather than shortened the communication of mathematical results. Communication required translation of symbols into the 'things' they represented." Pycior, *Symbols*, p. 145.
46 Hobbes, *English Works*, vol. 7, p. 330.
47 Ibid., vol. 7, p. 248.
48 Ibid., vol. 7, pp. 187–188.
49 Ibid., vol. 7, p. 184.
50 Ibid., vol. 7, p. 192.
51 Jesseph, *Squaring the Circle*, pp. 220–221. Jesseph is referring explicitly to Antonio Perez-Ramos's idea of maker's knowledge developed in Antonio Perez-Ramos, *Francis Bacon's Idea of Science and the Maker's Knowledge Tradition* (Oxford: Clarendon Press, 1988). I accept the term, however, with reservation, because I do not necessarily place Hobbes's 'constructivist' epistemology in the tradition of artisinal craftsmanship Perez-Ramos assigns to Bacon.
52 Jesseph, *Squaring the Circle*, pp. 241–242.
53 Pycior, *Symbols*, p. 135.
54 David Fate Norton, "The Myth of 'British Empiricism,'" *History of European Ideas*, 1 (1981), 331–334.
55 Richard W.F. Kroll, "The Question of Locke's Relation to Gassendi," *Journal of the History of Ideas*, 15 (1984), 339–359.
56 Ibid., p. 341.
57 Ibid., p. 346. Boyle makes this claim in the introduction to *The Origine of Formes and Qualities* (Oxford, 1666).
58 Kroll, p. 346.
59 Ibid., p. 350.

60 Fred S. Michael and Emily Michael, "The Theory of Ideas in Gassendi and Locke," *Journal of the History of Ideas* 51 (1990), 379–399.

61 See Thomas Stanley, *The History of Philosophy, Containing the Lives, Opinions, Actions and Discourses of the Philosophers of every Sect*, 3rd edn. (London, 1701), p. 545ff.

62 Michael and Michael, "Theory of Ideas," p. 380.

63 Ibid., p. 383. This, of course, does not prove that Locke *did* read Gassendi in the original, but I think it should remove any hesitation to at least consider the possibility.

64 On this subject, see G. A. J. Rogers, "Contexte des rapports intellectuels entre Hobbes et Locke," in *Hobbes et Locke: Archives de Philosophie* 55 (1992), and *Locke's Enlightenment: Aspects of the Origin, Nature and Impact of His Philosophy* (Hildesheim: G. Olms, 1998).

65 John Locke, *An Essay Concerning Human Understanding*, 4th edn. (1700) with critical notes by Peter H. Nidditch (Oxford: Clarendon Press, 1975; reprint: 1985), I, I, 8, p. 47. Michael Ayers suggests that Locke is modeling his notion of an 'idea' on Gassendi's own usage, and cites the similar passage in the *Syntagma philosophicum* where Gassendi comments that he uses the term because it is "familiar and well-worn." Ayers, *Locke: Epistemology and Ontology* (London: Routledge, 1996), vol. 1, p. 45.

66 Locke, *Essay*, Book. 1, Chapter 2, §1, p. 48.

67 Locke, *Essay*, Book. 1, Chapter 2, §15, p. 55.

68 Locke, *Essay*, Book. 2, Chapter 1, §2, p. 104 and Book. 2, Chapter 1, §23, p. 117.

69 Locke, *Essay*, Book. 2, Chapter 11, §6, p. 158.

70 Locke, *Essay*, Book. 2, Chapter 8, §7, p. 134.

71 Locke, *Essay*, Book. 2, Chapter 11, §9, p. 159.

72 Locke, *Essay*, Book. 2, Chapter 23, §32, p. 313.

73 Locke, *Essay*, Book. 2, Chapter 32, §1, pp. 384–385. There is some question about whether Locke intended ideas to be 'mental images' formed from experience, and Peter Alexander claims that there is no evidence that Locke "takes ideas *as concepts* to be mental images or that he supposes that we do our abstract thinking in pictures." Peter Alexander, *Ideas, Qualities and Corpuscles: Locke and Boyle on the External World* (Cambridge: Cambridge University Press, 1985), p. 96. Alexander suggests that Locke italicized the word 'Idea' because it was a literal translation of the Greek word originally meaning 'form' or 'pattern,' from which the Latin term 'species' was also derived.

74 Ayers, *Locke*, vol. 1, p. 50.

75 Locke, *Essay*, Book. 3, Chapter 2, §8, p. 408.

76 Locke, *Essay*, Book. 3, Chapter 3, §6, pp. 410–411 and Book. 3, Chapter 3, §11, p. 414.

77 Locke, *Essay*, Book. 3, Chapter 6, §6, p. 442.

78 Locke, *Essay*, Book. 3, Chapter 6, §37, p. 462.

79 Locke, *Essay*, Book. 1, Chapter 2, §8, p. 51 and Book. 1, Chapter 2, §16, p. 56.

80 Locke, *Essay*, Book. 1, Chapter 2, §28, p. 65; also Book. 2, Chapter 16, §6, p. 207.

81 Locke, *Essay*, Book. 2, Chapter 16, §5–§6, pp. 205–207.

82 Locke, *Essay*, Book. 2, Chapter 16, §7, p. 208.

83 Locke, *Essay*, Book. 3, Chapter 4, §19, p. 419–420.

84 Locke, *Essay*, Book. 4, Chapter 1, §9, p. 530 and Book. 4, Chapter 2, §9, p. 534.

85 Locke, *Essay*, Book. 4, Chapter 4, §6, p. 565.

86 Locke, *Essay*, Book. 4, Chapter 4, §8, p. 566.

87 Douglas Jesseph states that Berkeley's challenge "was taken seriously and set much of the agenda for British mathematics for more than two decades," *Berkeley's Philosophy of Mathematics* (Chicago: University of Chicago Press, 1993), p. 299, while Pycior maintains that British mathematicians allowed Berkeley's critique in *The Analyst* to "largely set their research program early in the second third of the eighteenth century," *Symbols*, p. 240.

88 Pycior, *Symbols*, p. 212.

89 George Berkeley, *George Berkeley's Commonplace Book*, ed. G. A. Johnston (London: Faber & Faber, 1930). For example, Berkeley writes that Hobbes's opinion "that existence was

distinct from perception [was] of horrible consequence" (§811), and calls Hobbes, along with Epicurus and Spinoza, "a declared enemy of religion" (§836).

90 Berkeley, *Commonplace Book*, §823, p. 99; George Berkeley, *Mathematical Miscellanies*, in *The Works of George Berkeley, D.D., Bishop of Cloyne*, ed. George Sampson (London: George Bell & Sons, 1897), vol. 1, p. 58.

91 The above references support Pycior's contention that although "Berkeley would eventually disagree with Locke . . . Locke's *Essay* nevertheless exercised a formative influence on him," noting particularly that "Locke's thoughts on ideas, number, and algebra were extremely important." Pycior, *Symbols*, p. 214.

92 Berkeley, *Commonplace Book*, esp. §272, §346, §377, §461, §504 and §513.

93 Pycior, *Symbols*, p. 209.

94 There is an extensive secondary literature on Berkeley's general philosophy and philosophy of immaterialism, but until recently not much on Berkeley's philosophy of science and mathematics. A small sampling of works on Berkeley's general theory includes A. A. Luce, *Berkeley and Malebranche: A Study in the Origins of Berkeley's Thought* (Oxford: Clarendon Press, [1934] 1967), an older but important work that describes the influence of Malebranche's thought on the development of Berkeley's philosophy; A. C. Grayling, *Berkeley: The Central Arguments* (London: Duckworth, 1986), George Pitcher, *Berkeley* (London: Routledge, 1977) and Kenneth P. Winkler, *Berkeley: An Interpretation* (Oxford: Clarendon Press, 1989); and J. O. Urmson's *Berkeley* (Oxford: Oxford University Press, 1982), and Gabriel Moked, *Particles and Ideas: Bishop Berkeley's Corpuscularian Philosophy* (Oxford: Clarendon Press, 1988) are general works that deal explicitly with Berkeley's philosophy of science and mathematics, although Urmson in particular is quite cursory in his treatment of the mathematics. As far as I am aware, Douglas M. Jesseph's *Berkeley's Philosophy of Mathematics* is the only book-length study of Berkeley's mathematics available, and luckily it is an excellent one, although Pycior's *Symbols* includes a very good chapter on Berkeley, and both Pycior and Jesseph have articles on Berkeley's philosophy of mathematics: see Helena M. Pycior, "Mathematics and Philosophy: Wallis, Hobbes, Barrow, and Berkeley," *Journal of the History of Ideas* 48 (1987), 65–86; and Douglas M. Jesseph, "Berkeley's Philosophy of Geometry," *Archiv für Geschichte der Philosophie* 72 (1990), 301–322.

95 I will focus on four principal texts in this section: the revised (1734) edition of the *Principles*, the *Essay towards a New Theory of Vision* (henceforth '*New Theory*'), *De Motu*, and *The Analyst*, which roughly correspond to the three major topics I will investigate: the *Principles* develops Berkeley's philosophy of immaterialism and theory of signification, the *New Theory* presents his beliefs about the 'visual language' of nature, and *De Motu* and *The Analyst* concern his philosophy of mathematics.

My text for the *Principles* comes from *The Works of George Berkeley*, ed. A. A. Luce and T. E. Jessop (London: Thomas Nelson & Sons, 1949), vol. 2. This is the best collection of Berkeley's complete works available, but for his more specialized treatises I have relied on critical editions: the *New Theory* is reproduced in Colin M. Turbayne (ed.) *Works on Vision* (Indianapolis: Bobbs-Merrill, 1963), while Douglas M. Jesseph provides a translation of *De Motu* and a critical text of *The Analyst* in *De Motu and The Analyst*, New Synthese Historical Library, vol. 41 (Dordrecht: Kluwer Academic, 1992). All references will be to page and corresponding section numbers. Berkeley, *Principles*, Introduction, §15, pp. 33–34.

96 Berkeley, *Principles*, I, §6, p. 43.

97 Ibid., I, §5, p. 43.

98 Ibid., I, §23, p. 50.

99 Berkeley, *New Theory of Vision*, §3, p. 20.

100 Ibid., §9, p. 21.

101 Ibid., §73, p. 52.

102 Ibid., §77, p. 58.

103 Ibid., §123, p. 78:

> Now, I do not find that I can perceive, imagine, or anywise frame in my mind such an abstract object as is here spoken of. A line or surface which is neither black, nor white, nor blue, nor yellow, etc., nor long, nor short, nor rough, nor smooth, nor square, nor round, etc., is perfectly incomprehensible. This I am sure of as to myself; how far the faculties of other men may reach they best can tell.

104 Ibid., §125, p. 79.

105 Ibid., §130, p. 83.

106 Ibid., §140, p. 88.

107 Ibid., §144, p. 90.

108 Ibid., §147, pp. 91–92.

109 Berkeley, *Theory of Vision, or Visual Language*, in *Works on Vision*, §40, p. 138. Note the implied voluntarism in this statement.

110 Berkeley's philosophy here is strikingly similar to approaches in twentieth-century linguistic structuralism, where meaning is conditioned by deep structures built into language, and not by necessary relationships between signs and objects signified.

111 Jesseph, *Berkeley's Philosophy of Mathematics*, p. 297.

112 ibid., p. 4.

113 Berkeley, *Principles*, §12, p. 46; *Commonplace Book*, §569, p. 67.

114 Berkeley, *Principles*, §12, p. 46.

115 Berkeley, *New Theory*, §109, pp. 71–72.

116 Berkeley, *Principles*, §120, p. 96.

117 Ibid., p. 97.

118 Jesseph does not hesitate to call Berkeley's philosophy of arithmetic "formalist," and even goes so far as to propose Berkeley as the "first in [a] line of formalists" in a tradition "which antedates Hilbert." Jesseph, *Berkeley's Philosophy of Mathematics*, p. 106. Without addressing the question of priority in the history of mathematical formalism, I am much in agreement with Jesseph's characterization and find that Berkeley's approach simply amplifies formal tendencies that have been present in many of the earlier philosophies we have considered.

119 Berkeley, *New Theory of Vision*, §124, p. 79.

120 Berkeley writes that, "Every particular finite extension, which may possibly be the object of our thought, is an idea existing only in the mind." *Principles*, §124, p. 98.

4 Three Mathematicians: Constructivist Epistemology and the New Mathematical Methods

1 Michel Blay, *Reasoning with the Infinite* (Chicago: University of Chicago Press, 1998, p. 3.

2 Ted Honderich (ed.) *The Oxford Companion to Philosophy* (Oxford: Oxford University Press, 1995), pp. 159–160.

3 The definition adds that "philosophical opponents of constructivism include realists, who hold that since mathematical objects exist independent of the mind of the mathematician, one can establish the existence of an object without showing how to find it." *Oxford Companion*, p. 160.

4 Peter Dear, *Discipline and Experience: The Mathematical Way in the Scientific Revolution* (Chicago: University of Chicago Press, 1995), pp. 8–9.

5 There are a number of excellent sources on the history of algebra and analysis. A few of the most important are Carl B. Boyer, *A History of Analytic Geometry* (New York: Scripta Mathematica, 1956) and *A History of the Calculus and its Conceptual Development* (New York: Dover, 1959); Florian Cajori, *A History of Mathematical Notations*, 2 vols. (Chicago: Open Court, 1929); Jacob Klein, *Greek Mathematical Thought and the Origin of Algebra* (Cambridge, MA; MIT Press, 1968; reprint, New York: Dover, 1992); and Morris Klein, *Mathematics from Ancient to Modern Times* (New York: Oxford University Press, 1972). More recent contributions include Henk Bos, *Redefining Mathematical*

Exactness: Descartes' Transformation of the Early Modern Concept of Construction (New York: Springer Verlag, 2001), Blay, *Reasoning*; Katherine Neal, *From Discrete to Continuous: The Broadening of Number Concepts in Early Modern England* (Dordrecht: Kluwer Academic Publishers, 2002), and Helena M. Pycior, *Symbols, Impossible Numbers and Geometric Entanglements: British Algebra through the Commentaries on Newton's 'Universal Arithmetick'* (Cambridge: Cambridge University Press, 1997).

6 Pycior, Katherine Hill, and Jesseph, for example, have stressed this point. See Pycior, *Symbols*, Katherine Hill, "Neither Ancient nor Modern: Wallis and Barrow on the Composition of Continua," pt. 1, *Notes and Records of the Royal Society of London* 50 (1996), pp. 165–168, and Douglas Jesseph, *Berkeley's Philosophy of Mathematics* (Chicago: University of Chicago Press, 1993), and *Squaring the Circle: The War between Hobbes and Wallis* (Chicago: University of Chicago Press, 1999).

7 Perhaps the most prominent early eighteenth-century example is George Berkeley's critique of Newtonian calculus in his treatise *The Analyst*, in *De Motu and The Analyst*, trans. Douglas M. Jesseph (Dordrecht: Kluwer Academic Publishers, 1992). As Jesseph has argued, Berkeley's objection to the method of fluxions has a strongly nominalistic flavor. Jesseph, *Berkeley's Philosophy of Mathematics*, pp. 94–95 and 118–120.

8 Klein, *Greek Mathematical Thought*, p. 175. Emphasis in original.

9 Ibid., p. 175. It should be noted that Katherine Neal's recent monograph *From Discrete to Continuous*, challenges Klein's history of the 'symbolic' understanding of number. Neal argues, for example, that Klein's "big picture orientation" confuses the distinction between abstract and geometrical magnitudes, which Neal considers the central point of debate in early modern mathematics. Neal, *From Discrete to Continuous*, pp. 4–5.

10 All of Wallis' major mathematical works, including *Mathesis Universalis* and *Arithmetica Infinitorum*, are collected in his *Opera Mathematica* of 1693–1699 but were originally published in 1656 and 1657. John Wallis, *Opera Mathematica*, 3 vols. (Oxford, 1693–1699; facsimile reprint, Hildscheim: Georg Olms Verlag, 1972). On Wallis' importance to the development of the calculus and seventeenth-century mathematics in general, see Pycior, *Symbols*; Klein, *Greek Mathematical Thought*; and Boyer, *The History of the Calculus*.

11 Wallis, as did Barrow, turned increasingly towards theological subjects later in life. His many religious works include: *A Defense of the Christian Sabbath. In Answer to a Treatise of Mr. Tho. Bampfield Pleading for Saturday-Sabbath* (London, 1692); *Theological Discourses: Containing viii Letters and iii Sermons Concerning the Blessed Trinity* (London, 1692); *Sermons; Now First Printed from the Original Manuscripts of John Wallis, D.D. Some Time Savilian Professor of Geometry in the University of Oxford, Keeper of the Archives, Member of the Royal Society, and Chaplain Ordinary to King Charles II. To Which are Prefixed, Memoirs of the Author, with some Original Anecdotes; and a Recommendatory Introduction, by the Rev. C.E. De Coetlogin, M.A.* ed. W. Wallis (London, 1791).

12 Robert Grevile Brooke, *The Nature of Truth, its Union and Unity with the Soule* (London, 1641).

13 John Wallis, *Truth Tried: or, Animadversions on a Treatise Published by the Right Honourable Robert Lord Brook, Entitled, The Nature of Truth, Its Union and Unity with the Soule. Which (saith he) is One in its Essence, Faculties, Acts; One with Truth* (London, 1642).

14 John Wallis, "The Autobiography of John Wallis, F.R.S.," ed. Christoph J. Scriba, *Notes and Records of the Royal Society* 25 (1970), 29.

15 Wallis, "Autobiography," pp. 40–41.

16 Wallis, *Truth Tried*, pp. 37, 93.

17 Ibid., p. 37.

18 Ibid., p. 3.

19 Ibid., p. 49.

20 Ibid., p. 84.

21 Ibid., pp. 83–84.

22 Pycior, *Symbols*, p. 101.

23 Richard Foster Jones, *Ancients and Moderns: A Study of the Rise of the Scientific Movement in Seventeenth-Century England* (St. Louis: Washington University Press, 1961; reprint, New York: Dover, 1982), cf. pp. 128–130, 255–260.

24 Hill, "Neither Ancient nor Modern," p. 171.

25 Wallis, *Truth Tried*, p. 37.

26 Ibid., p. 40.

27 John Wallis, *Opera*, p. 21.

28 John Wallis, *A Treatise of Algebra*, pp. 298–299.

29 Ibid., p. 1.

30 Ibid., p. 305.

31 Ibid., p. 68.

32 Wallis, *Truth Tried*, pp. 83–84.

33 Wallis, *A Treatise of Algebra*, pp. 264–265.

34 It has often been remarked by historians that the 'narrative' approach to describing nature characterized early experimental natural philosophy. For example, Boyle's narrative descriptions of experimental events have been described by Steven Shapin as a kind of "virtual witnessing." I am suggesting that this epistemological tendency in experimental philosophy can be extended to an empirical philosophy of mathematics. See Steven Shapin, "Pump and Circumstance: Robert Boyle's Literary Technology," *Social Studies of Science* 14: 481–520, and *A Social History of Truth: Civility and Science in Seventeenth-Century England* (Chicago: University of Chicago Press, 1994), especially Chapters 5 and 7.

35 Wallis, *A Treatise of Algebra*, p. 292.

36 Ibid., p. 292.

37 Ibid., pp. 281–282.

38 Ibid., p. 298.

39 Ibid., p. 308.

40 Hill, "Neither Ancient nor Modern," p. 165.

41 Blay, *Reasoning*, p. 4.

42 Wallis to Oldenburg, 5 December, 1668, *The Correspondence of Henry Oldenburg*, 13 vols., ed. A. Rupert Hall and Marie Boas Hall (Madison, WI: University of Wisconsin Press, 1965–1973), vol. V, p. 221.

43 Michael S. Mahoney, "Barrow's Mathematics: Between Ancients and Moderns," in Mordechai Feingold (ed.) *Before Newton: The Life and Times of Isaac Barrow* (Cambridge: Cambridge University Press, 1990), p. 181.

44 J. M. Child, *The Geometrical Lectures of Isaac Barrow: Translated, with Notes and Proofs, and a Discussion on the Advance made Therein on the Work of His Predecessors in the Infinitesimal Calculus* (Chicago: Open Court, 1916), pp. vii–xi. More fully elaborated in J. M. Child "The 'Lectiones Geometricae' of Isaac Barrow," *The Monist* 26 (1916), 251–267.

45 Mahoney, "Barrow's Mathematics," pp. 180–181. For the debate on Barrow's contribution to the calculus, see also Florian Cajori "Who Was the First Inventor of the Calculus?," *American Mathematical Monthly* 26 (1919), 15–20; Margaret E. Baron, *Origins of the Infinitesimal Calculus* (Oxford: Oxford University Press, 1969); and D. T. Whiteside's "General Introduction," in vol. 1 of *The Mathematical Papers of Isaac Newton*, 8 vols. (Cambridge: Cambridge University Press, 1966–1983). Feingold, however, argues for a more generous reading of Barrow's influence on Newton, partly on the grounds that "no present-day verdict on Barrow's published work vis-à-vis the new mathematics can serve as the sole basis for embracing his learning and appreciating his stature in his own time." Mordechai Feingold, "Newton, Leibniz, and Barrow Too: An Attempt at a Reinterpretation," *Isis* 84 (1993), 337.

46 Mordechai Feingold, "Isaac Barrow: Divine, Scholar, Mathematician," in Mordechai Feingold (ed.) *Before Newton: The Life and Times of Isaac Barrow* (Cambridge: Cambridge University Press, 1990), pp. 1–104, esp. pp. 53–67.

47 Isaac Barrow, *The Usefulness of Mathematical Learning explained and demonstrated: Being Mathematical Lectures Read in the Publick Schools at the University of Cambridge*, trans. John

Kirkby (London, 1734; facsimile reprint, London: Frank Cass & Co., 1970); *Isaac Barrow's Optical Lectures (Lectiones XVIII)*, trans. H. C. Fay, ed. A. G. Bennett and F. F. Edgar (London: 1667; reprint, London: The Worshipful Company of Spectacle Makers, 1987).

48 Barrow, *Usefulness*, p. xxx.

49 Pycior, *Symbols*, p. 165.

50 Barrow, *Usefulness*, p. xxviii.

51 Ibid., p. 113.

52 Ibid., p. 115; Wallis, *Truth Tried*, p. 49.

53 Barrow, *Usefulness*, p. 115. Emphasis added.

54 Pycior also argues that Hobbes and Barrow should be viewed together in this light: "the empiricist inclinations of the English Scientific Revolution affected mathematics, with Hobbes and Barrow arguing for the primacy of sense-based geometry over arithmetic and algebra." *Symbols*, p. 166.

55 Barrow, *Usefulness*, p. 30.

56 Ibid., p. 29.

57 Ibid., p. 19.

58 Ibid., p. 28.

59 Ibid., pp. 34–35.

60 Ibid., p. 41.

61 Ibid., pp. 36–37.

62 Ibid., p. 76. I am convinced he is referring to Gassendi here, whom he cites positively in his Preface (p. xxvii).

63 Barrow, *Geometrical Lectures*, p. 49.

64 Barrow, *Usefulness*, pp. 65–66.

65 Ibid., p. 90.

66 Ibid., p. 113.

67 Ibid., p. 53.

68 Ibid., p. 69.

69 Ibid., pp. 72–75.

70 Ibid., p. 73.

71 Antoni Malet, "Isaac Barrow on the Mathematization of Nature: Theological Voluntarism and the Rise of Geometrical Optics," *Journal of the History of Ideas* 58 (1987), pp. 277–278.

72 Barrow, *Usefulness*, p. 111.

73 Margaret Osler, *Divine Will and the Mechanical Philosophy: Gassendi and Descartes on Contingency and Necessity in the Created World* (Cambridge: Cambridge University Press, 1994); and "The Intellectual Sources of Robert Boyle's Philosophy of Nature: Gassendi's Voluntarism and Boyle's Physico-Theological Project," in Gary Kroll (ed.) *Philosophy, Science and Religion*, pp. 178–198; Malet, "Isaac Barrow"; and Edward B. Davis, "Rationalism, Voluntarism, and Seventeenth-Century Science," in Jitse M. van der Meer (ed.) *Facets of Faith and Science* vol. 3: *The Role of Beliefs in the Natural Sciences* (Lanham, MD: University Press of America, 1996).

74 Barrow, *Usefulness*, pp. 88–89.

75 Ibid., p. 111.

76 Ibid., pp. 111–112.

77 Malet, "Isaac Barrow," p. 279.

78 Barrow, *Usefulness*, p. 169.

79 Ibid., p. 116.

80 Malet, "Isaac Barrow," p. 277.

81 Hill, "Neither Ancient nor Modern," p. 172. She goes on to claim that "this view was not a product of hostility to modern methods in mathematics, as previous historians have often assumed."

82 Pycior explicitly connects the two philosophers: "the empiricist inclinations of the English Scientific Revolution affected mathematics, with Hobbes and Barrow arguing

for the primacy of the sense-based geometry over arithmetic and algebra," *Symbols*, p. 166.

83 That Barrow was familiar with Gassendi's work is beyond dispute, but the extent to which Gassendi influenced Barrow is an open question. Mordechai Feingold is of the opinion that Barrow "was very well read in Gassendi," and notes that the reason only one of Gassendi's works appears in Barrow's library (*Institutio astronomicae*) is because Barrow sold his 'student library' before his trip to the continent in 1655 (Feingold, personal communication). Evidence that Barrow had a wider knowledge of Gassendi can be found in his MA oration, in which he groups Gassendi among a set of modern "men of outstanding genius and learning." See Isaac Barrow, "MA Oration on Cartesian Philosophy," trans. Ian Stewart, in "Fleshy Books: Isaac Barrow and the Oratorical Critique of Cartesian Natural Philosophy," *History of Universities* 16 (2000), 35–102. Barrow repeatedly cites Gassendi approvingly in this oration, and he advocates support for a Gassendist/Epicurean ontology of atoms and void (p. 80). Barrow does not name individual works in most cases, but at one point mentions an argument "noted by Gassendi in his life of Epicurus," which indicates that he had read Gassendi's *De vita et moribus Epicuri*, which was first published in 1647 but which was expanded and included with his 1649 treatise on atomism, *Animadversiones in decimum librum Diogenis Laertii, qui est de vita, moribus, plastique Epicuri*. There is no way of establishing which version Barrow read, but the *Animadversiones* was certainly in circulation in England in the early 1650s (Charleton's 1654 *Physiologia* is based on it), and Barrow's arguments concerning atoms and vacuum throughout the oration suggest he was familiar with the ideas presented in the latter treatise.

84 I. Bernhard Cohen, *The Newtonian Revolution* (Cambridge: Cambridge University Press, 1980), p. 30.

85 Mahoney, "Mathematization," in Daniel Garber and Michael Ayers (eds.) *The Cambridge History of Seventeenth-Century Philosophy*, vol. 1 (Cambridge: Cambridge University Press, 1998), pp. 749–750.

86 Dear, *Discipline*, p. 8. It would be an unwarranted stretch, however, to claim that Newton shared Gassendi's radical nominalistic conviction that abstract representations are always illegitimate or fictitious. Gassendi's position derives largely from his conflation of mathematics, language, and formal logic, a connection which Newton never explicitly pursued or noted.

87 The source for this is Newton's manuscript *Quaestiones quaedam philosophicae*, which was written between 1664–1665. It has been edited and reprinted in J. E. McGuire and Martin Tamny, *Certain Philosophical Questions: Newton's Trinity Notebook* (Cambridge: Cambridge University Press, 1983).

88 McGuire and Tamny, *Certain Philosophical Questions*, pp. 98–99. "The pattern of Newton's argument indicates that he wishes to minimize the gap between the mathematicals and the perceptible quantity of physical objects."

89 Ibid., pp. 108–112.

90 As Newton explains in his final draft of the 1704 *De quadratura curvarum*. Isaac Newton, *The Mathematical Papers of Isaac Newton*, 8 vols., ed. and trans. D. T. Whiteside *et al*. (Cambridge: Cambridge University Press, 1967–1981), vol. 8, p. 123:

> Mathematical quantities I here consider not as consisting of least possible parts, but as described by continuous motion. Lines are described and by describing generated not through the apposition of parts but through the continuous motion of points; surface-areas are through the motions of lines, solids through the motion of surface-areas, angles through the rotation of sides, times through continuous flux, and the like in other cases. These geneses take place in the reality of physical nature and are daily witnessed in the motion of bodies.

91 Although McGuire and Tamny note that Newton was critical of Hobbes, they also suggest that—particularly on the subject of optics—Newton was influenced by Hobbes's epistemology. McGuire and Tamny, *Certain Philosophical* Questions, pp. 216ff.

92 Niccolò Guicciardini, *Reading the Principia: The Debate on Newton's Mathematical Methods for Natural Philosophy from 1687 to 1736* (Cambridge: Cambridge University Press, 1999), pp. 6–7.

93 Ibid., p. 159.

94 This is, of course, leaving aside the question of *metaphysical* justification for formalism, on which subject Leibniz and Wallis were not closely aligned.

95 Newton, *Mathematical Papers*, vol. V, p. 471.

96 Newton, *Mathematical Papers*, vol. V, p. 477.

97 Guicciardini, *Reading*, p. 167.

98 Draft of "Geometry," in Newton, *Mathematical Papers*, vol. 8, p. 289. Whiteside notes that there is no certain date for this unpublished work, but that it had probably been written by 1693.

99 Dear, *Discipline*, p. 212.

100 Newton, *Philosophiae Naturalis Principia Mathematica, Mathematical Principles of Natural Philosophy*, trans. I. Bernard Cohen and Anne Whitman (Berkeley, CA: University of California Press, 1999), p. 943. Newton voiced a similar sentiment in a letter to Henry Oldenburg concerning mathematical optics. Newton to Oldenburg, 11 June, 1672, in Newton, *Correspondence*, vol. 1, p. 187:

> I said indeed that the Science of colors was Mathematicall & as certain as any other part of Optiques; but who knows not that Optiques & many other Mathematicall Sciences depend as well on Physicall Principles as on Mathematicall Demonstrations: And the absolute certainty of a Science cannot exceed the certainty of its principles.

101 Thomas Kuhn, *The Structure of Scientific Revolutions*, 2nd edn. (Chicago: University of Chicago Press, 1970).

102 See Ernan McMullin, *Newton on Matter and Activity* (Notre Dame, IN: Notre Dame University Press, 1978) for a discussion of Newton's concept of force as an active principle in matter. Also, Cohen, "Introduction" to the *Principia*; Keith Hutchison, "What Happened to Occult Qualities in the Scientific Revolution?," *Isis*, 73 (1982), 233–253; John Henry, "Occult Qualities and the Experimental Philosophy: Active Principles in Pre-Newtonian Matter Theory," *History of Science*, 26 (1986), 335–381.

103 Kuhn, *Structure*, pp. 67–68. I use the term 'crisis' in a very limited sense, and without assuming the general framework for revolutionary change posed in *Structure*. It does seem, however, that the late seventeenth-century uncertainty about which mathematical methods and mechanical model should be adopted well fits Kuhn's description of "professional insecurity" that ultimately leads to new consensus.

104 The best account of this conflict is Guicciardini, *Reading*.

105 Berkeley, *The Analyst*, §8, p. 169.

106 Ibid., §20, pp. 180–181.

107 Jesseph, *Berkeley's Philosophy of Mathematics*, p. 106.

108 Berkeley, *The Analyst*, §20, p. 180.

109 Ibid., §35, p. 199.

110 Ibid., §49, p. 208. For a thorough explication of Berkeley's technical critique of fluxions and their applications, see Jesseph, "Editor's Introduction" to *The Analyst*, in *De Motu and The Analyst*, pp. 111–145.

111 Berkeley, *The Analyst*, §14, p. 174.

112 Ibid., §15, pp. 175–176.

113 See, Jesseph, *Berkeley's Philosophy of Mathematics*, Guicciardini, *Reading*, and Pycior, *Symbols*.

114 Berkeley, *The Analyst*, §15, p. 176: "Whether you argue in Symbols or in Words, the Rules of right Reason are still the same. Nor can it be supposed, you will plead a Privilege in Mathematics to be exempt from them."

115 Ibid., §22, p. 183.

116 Geoffry Cantor, "Berkeley's *The Analyst* Revisited," *Isis* 75 (1984), 669. Cantor focuses on the religious aspects of *The Analyst*, which he regards as essentially an attack on freethinkers, and characterizes Berkeley's mathematical critique as the voice of 'High Church Tory' theology applied to mathematics. Given the obvious and fairly indisputable links between Berkeley's mathematical philosophy in *The Analyst* and his earlier statements about representation in *Principles* and other works, I find Cantor's interpretation limited and hard to sustain. Berkeley may well have objected to freethinking, but the philosophy of mathematics was not simply a vehicle for Berkeley to pursue some grander religious 'agenda.'

117 Jesseph, *Berkeley's Philosophy of Mathematics*, p. 182, note 5.

118 Jesseph writes that "Berkeley's logical objections to the calculus are both brilliantly argued and devastatingly effictive," "Introduction," to *The Analyst*, p. 139, while Pycior calls *The Analyst* "one of the most influential mathematico-philosophical works ever published," *Symbols*, p. 240.

119 Berkeley, *De Motu*, §35, p. 89.

120 Ibid., §17, p. 80, and §28, p. 85, for example.

121 Ibid., §39, p. 90.

122 Ibid., §40, p. 91; §70, p. 106.

123 Instrumentalism, briefly, is the belief that mathematical demonstrations are prized for their utility, and not because they convey objective truth about physical reality. Jesseph, *Berkeley's Philosophy of Mathematics*, pp. 76–77.

124 Berkeley, *Theory of Vision*, in *Works on Vision*, ed. Colin M. Turbayne (Indianapolis: Bobbs-Merrill, 1963), §40, p. 138.

125 Pycior notes, for example, that in Berkeley's published exchange with James Jurin, the two "talked at each other, but not to each other." *Symbols*, p. 239.

126 Philalethes Cantabrigiensis [James Jurin], *Geometry No Friend to Infidelity: Or, A Defence of Sir Isaac Newton and the British Mathematicians, in a Letter to the Author of the "Analyst"* (London: T. Cooper, 1734).

127 George Berkeley, *A Defence of Free-thinking in Mathematics* (1735), in *The Works of George Berkeley, Bishop of Cloyne*, ed. A. A. Luce and T. E. Jessop (London: Thomas Nelson and Sons, 1948–1957), vol..4, pp. 109–141. Jesseph considers Jurin's attack to have been mostly ineffective stating that his response, "hardly comes up to the mark" and suggesting that in a subsequent response, *The Minute Mathematician*, "Jurin has in effect conceded defeat." Jesseph, *Berkeley's Philosophy of Mathematics*, pp. 238 and 247; Philalethes Cantabrigiensis [James Jurin], *The Minute Mathematician: or, the Free-Thinker no Just-Thinker* (London: T. Cooper, 1735).

128 Pycior, *Symbols*, p. 241.

129 David Gregory, *The Elements of Physical and Geometrical Astronomy: By David Gregory. To which is Annex'd, Dr. Halley's Synopsis of the Astronomy of Comets* (London, 1726; facsimile reprint, New York, Johnson Reprint Co., 1972), 2 vols.; Colin Maclaurin, *An Account of Sir Isaac Newton's Philosophical Discoveries* (London, 1748; facsimile reprint, New York: Johnson Reprint Co., 1968); Henry Pemberton, *A View of Sir Isaac Newton's Philosophy* (London, 1728; facsimile reprint, New York, Johnson Reprint Co., 1972). Samuel Clarke's work consists of a very lengthy commentary on (and in effect correction of) Jacques Rohault's Cartesian treatise, *A System of Natural Philosophy*, ed. and trans. Samuel Clarke (London, 1723; facsimile reprint, New York: Johnson Reprint Co., 1969), 2 vols.

130 Colin Maclaurin, *A Treatise of Fluxions, in Two Volumes* (Edinburgh: 1742; 2nd edn., London: 1801), 2 vols. See Erik Lars Sangeng's unpublished dissertation "Colin Maclaurin

and the Foundations of the Method of Fluxions (Calculus)," (Ph.D. diss., Princeton University, 1990) for a detailed study of Maclaurin's mathematical method.

131 Indeed, Jesseph goes even further (and perhaps too far), claiming that far from being "a refutation of Berkeley," *Fluxions* was "a comprehensive treatise in the calculus with a Berkeleyan basis" that "essentially grants Berkeley's case against the Newtonian and Leibnizian methods and proceeds from there." Jesseph, *Berkeley's Philosophy of Mathematics*, p. 285.

132 Maclaurin, *Fluxions*, vol. 1, pp. vii, viii.

133 Ibid., vol. 1, p. 2.

134 Ibid., vol. 1, pp. 38–39. For example, the infinite division of real magnitudes, the paradox of motion in a plenum, the existence of Cartesian vortices, etc.

135 Ibid., vol. 1, pp. 39–40.

136 Ibid., vol. 1, pp. 3, 47.

137 Ibid., vol. 1, p. vii.

138 Ibid., vol. 1, p. 49.

139 Ibid., vol. 1, pp. 51–52.

140 Maclaurin, *Fluxions*, vol. 2, II, i, §697–98, pp. 163–164.

141 Maclaurin, *Account*, p. 8.

142 Ibid., p. 9:

> It is evident that, in mathematics, so in natural philosophy, the investigation of difficult things ought ever precede the method of composition, or the *synthesis*. For in any other way, we can never be sure that we assume the principles which really obtain in nature; and that our system, after we have composed it with great labour, is not mere dream and illusion.

This first sentence is a nearly verbatim paraphrase of Newton's statement in Query 31 of the *Opticks*.

143 Maclaurin, *Account*, p. 14.

144 Ibid., pp. 14, 19.

145 Ibid., p. 51.

146 Ibid., pp. 90–91, 221.

Conclusion

1 Paolo Mancosu, *Philosophy of Mathematics and Mathematical Practice in the 17th Century* (New York: Oxford University Press, 1996), pp. 10–12; Jacob Klein, *Greek Mathematical Thought and the Origin of Algebra*, trans. Eva Braun (Cambridge, MA: MIT Press, 1968).

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Index

Abelard, Peter 22

Abrégé de la Philosophie de Gassendi (Bernier, F.) 67

Academic skepticism 11, 18, 30, 138n32

Académie Française á Paris 54

An Account of Sir Isaac Newton's Philosophical Discoveries (Maclaurin, C.) 118, 120

Adams, Marilyn McCord 133n43

Against the Mathematicians (Sextus Empiricus) 18

algebra 7, 49–50, 55, 63–64, 74, 80, 87, 140n72, 145n91, 146–47n5; Barrow's approach to 99–101, 106; Berkeley's formalist approach to 113–14, 115, 116–17; English 'early modern' 90; Hobbes on 65–67; Maclaurin on 119–22; Newton and algebraic notation 109; as science of 'signs' 117–18; symbolic algebra of Vieta 49, 86, 126; Wallis' approach to 92–96

analysis 7, 83–84, 86, 93, 95, 98, 108, 110, 121; abstraction and 94, 99–100; algebra as 92; Barrow on 99–100, 106; Burtt's analysis of mataphysical basis for mathematics 6; discovery and 119; Funkenstein's analysis of knowledge by construction 9; Gassendi's analysis of logic in schools 35–36; geometrical analysis 63; as heuristic device 87, 106; Hobbes' view of 61; legitimacy of 86; mathematics as tool of 24; Proclus' analysis of philosophical issues 17–18; symbolic analysis 64, 92, 95, 113; synthesis and 29, 60, 121

The Analyst; or a Discourse Addressed to an Infidel Mathematician (Berkeley) 113, 115, 117, 118

Animadversiones in decimum librum Diogenis Laertii (Gassendi) 36, 40, 67, 68, 137n16

Apollonius 11, 17

Archimedes 11, 17, 84; geometry of 5

Ariew, Roger 136–37n8

Aristotle 5, 11–13, 15–17, 20, 22, 25–26, 28, 31, 37, 49; Aristotelian hierarchy 8; *Categories* 131n24; demonstrative science of 7; dissemination of texts in Middle Ages 130n1; linguistics of 29; logic of 17, 19, 29, 35, 36, 43; philosophy of 21, 29, 111–12; plenist system of 38; *Posterior Analytics* 131n24; rational syllogistic method 30; signification theory of 25; theory of 25, 124, 125

arithmetic 9, 14, 28, 49, 61, 63, 69, 71, 102, 117; Aristotle and primacy of 16; Barrow's nominalistic view on 98–101, 103, 106; Berkeley's view on 80–81, 113–14, 116; Maclaurin on 120, 122; Wallis' approach to 87, 93–95

Arithmetica Infinitorum (Wallis) 87, 93

Arnauld, Antoine 48, 95

astronomy 1, 17, 35, 42, 47, 54, 104, 133n40, 137n11

atomism 34, 47, 53, 123, 136n6; of Autrecourt 26–27; classical atomism 27–28, 33; Descartes on atoms 49; Epicurean atomism 33, 34, 36, 42–43, 56; Gassendi on atoms 38–39; nominalism and 27–28; physics of 34–39; revival in 17th Century 135n1

Augustine 20, 21, 28

Autrecourt, Nicholas of 22, 25–27, 134n63, 136n6

Ayers, Michael 71, 129n8, 130n17, 144n65, 150n85

Bacon, Francis 8, 35, 54

Barrow, Isaac 3, 7, 9–10, 33, 54, 82, 96–107, 110–11, 119, 123–24; algebra,

approach to 99–101, 106, 117, 122; on analysis 99–100, 106; Berkeley's references to 74–75, 80; constructivism of 83–84, 95, 107, 108, 125; constructivist skepticism of 116; empiricism, commitment to 98; epistemological constructivism of 85; epistemology of 98, 99, 103, 106; Gassendi and 150n83; geometry, constructivist approach to 73, 74, 96–107; God in philosophy of 104–5, 106–7; nominalistic view on arithmetic 98–101, 103, 106; number as relation, concept of 92; synthetical geometrical approach of 87
Berkeley, George 3, 5, 7, 10, 22, 53, 67, 73–82, 87, 117–22; on abstract objects 145–46n103; arithmetic, view on 80–81, 113–14, 116; Barrow, Berkeley's references to 74–75, 80; constructivism, approach to 83; dissenting voice against the calculus 112–15; empirical skepticism of 122–23; epistemological constructivism of 85; epistemology of 76, 80, 81, 116; '*Esse est percepti*' 75–76; formalist approach to algebra 113–14, 115, 116–17; geometry, views on 80, 81, 82, 113–14, 116; mathematical philosophy of 74, 80–82, 121–22, 152n116; nominalism of 67, 69, 76, 81, 122–23; nominalist/constructivism of 124; philosophy of mind 75–76; on physics as approximation 116; *Principles of Human Knowledge* 55, 75–76, 117; visual language of nature 76–80
Bernier, François 67, 69
Blay, Michel 7, 84–85, 95–96, 106, 130n18, 146n1
Bloch, O.R. 136–37n8
Boyle, Robert 8, 34, 67, 68–69, 69, 135n2, 148n34
Brooke, Robert Grevile 87, 89, 147n11
Buridan, Jean 9, 22, 133n45
Burley, Walter 9, 22, 133n45
Burtt, E.A. 3, 5–6, 8, 84, 128n4

Cantor, Geoffry 115, 152n116
Carre, Meyrick H. 21–22
Cartesian: coordinates 94; cosmology and ontology 118; mathematical realism 34; mechanical philosophy 111; rationalism 30, 120; *see also* Descartes, René
Catholic doctrine 20
Charleton, Walter 40–41, 67–68, 87, 104, 108, 136n7
Child, J.M. 96, 148n44
Christianity: Christianized Aristotelian philosophy 39; doctrine of 11, 21; theology of 20, 21
Cicero 44
Clarke, Samuel 37, 112, 118
Clavius, Christoph 30, 85, 108
Clerselier, Claude 50
Commentary on the First Book of Euclid's Elements (Proclus) 17, 18
Commonplace Book (Berkeley) 74
Computato Sive Logica (Hobbes) 142n25
constructivism 5, 10, 55, 76, 83, 106, 126; of Barrow 83–84, 107, 125; Barrow's constructivist approach to geometry 73, 74, 96–107; compound ideas 45; constructivist epistemology 55–56, 83; constructivist skepticism 116; core value in epistemology 56–57; epistemological constructivism 8, 9, 10, 54, 62, 66, 83, 85–87, 90, 103–4, 124, 125; of Gassendi 42–46, 47; of Hobbes 56–57, 62; linguistic constructivism 124, 125; in mathematical philosophy 126–27; mathematization and constructivism 84–87; mechanical constructivism 62, 66, 102; in Middle Ages 20–28; of Newton 107, 110; origins of 9; philosophical opponents of 146n3; in 17th Century natural philosophy 85; use of term 129n10; of Wallis 125
Copernicus 1, 4, 5, 104; systems of 47–48
Cratylus (Plato) 12
Croll, Oswald 42
Cudworth, Ralph 17
Cusa, Nicholas of 17

d'Ailly, Pierre 137n17
Daston, Lorraine 6, 130n16
De Corpore (Hobbes) 55, 56, 57, 61, 62, 108, 109
De Motu (Berkeley) 115–16
De motu impresso a motore translato (Gassendi) 35
De quadratura curvarum (Newton) 150n90
De rerum natura (Lucretius) 38
De revolutionibus (Copernicus) 4
Dear, Peter 4, 8, 85–86, 110, 146n4, 150n86
Dee, John 6, 42
A Defence of Free-thinking in Mathematics (Berkeley) 117, 152n127
Democritus 13, 27, 31, 33, 49, 52; atomism of 26

Descartes, René 5, 7, 14, 17, 54–55, 83, 108, 111, 120–21, 126; on abstraction 94; on atoms 49; existence and perception 75; Gassendi and 33–36, 38, 48–53, 56; geometry of 50–52, 62; Hobbes and 60; knowledge and intellect 70; logic, Gassendi and Descartes on 55; mathematical philosophy of 47; mathematical physics of 1; mathematization and constructivism 84; mechanical system of 37; metaphysics 50, 51; *mathesis universalis* 49–50; methodological differences with Gassendi 52–53; nature abstractable for 49; objections of Gassendi to Descartes' metaphysics 51–52; philosophy of 140n76; philosophy of mathematics 47; physics for 50–51; principles, understanding of 51–52; rationalistic philosophy of 69, 95

Dialogues Concerning Two New Sciences (Galileo) 63

Digby, Kenelm 26

Dijksterhuis, E.J. 3, 5, 6

Diogenes Laertius 13

Disquisito Metaphysica adversus Cartesium (Gassendi) 48

Dogmatism 19

du Prat, Abraham 56

Duns Scotus 9, 20, 22

Elements (Euclid) 17

Elements of Philosophy (Hobbes) 57; *see also De Corpore*

The Elements of Physical and Geometrical Astronomy (Gregory, D.) 118

Empedocles 138n22

empiricism 16, 36, 123; Barrow's commitment to 98; of Locke 67; modern empiricism 67; multiple versions of 126; skeptical empiricism 32; tradition of 69

Enlightenment 118, 122, 123, 127, 132n30, 138n32

Epicurean Syntagma (Gassendi) 68

Epicureans 11

Epicurus 13, 27, 33, 34, 37, 38, 39, 52, 68; atomism of 33, 34–39, 56; philosophy of 37

epistemology 2, 8, 120, 122, 123; Aquinas' 21; atomism and nominalism in 27; Autrecourt's 26; of Barrow 98, 99, 103, 106; of Berkeley 76, 80, 81, 116; central issues in 11; constructivism, core value 56–57; constructivist 55–56, 83; epistemological constructivism 8, 9, 10, 54, 62, 66, 83, 85–87, 90, 103–4, 124, 125; epistemological status of language 4–5; of Gassendi 34, 36, 39–42, 43, 44, 45, 53, 68; Hobbes' *Theory of Representation* 57–62; linguistic and logical questions in 28; Locke's epistemological position 69–73; logic, epistemological skepticism in 122; mathematical philosophy, epistemological uncertainty in 112; of mathematization 2, 6–7; of Newton 111; of nominalism 9–10; nominalism, core value 56–57; nominalism and epistemology, relationship between 83; nominalist 18, 26, 33, 87, 125; Ockham's 24; of physics 42; of Plato 13–14, 17, 21; principles of 54–55; of Ramus 29; rationalist 19, 29; realist 15; of representation 83; of Sanches 30–32; scientific 35, 57; 17th Century currency 54–55; of Wallis 95

An Essay Concerning Human Understanding (Locke) 55, 68, 69–73, 120, 144n65

Essay Towards a New Theory of Vision (Berkeley) 76, 80–81

Euclid 11, 17, 18; geometry of 5, 17, 86, 96, 98, 110

Exercitationes paradoxicae adversus Aristoteleos (Gassendi) 36, 39, 42–44, 46, 47

Faur de Pibrac, Henri du 35, 137n12

Feingold, Mordechai 97, 148n45

Fermat, Pierre de 93

Fisher, Saul 136n5

Funkenstein, Amos 9, 56, 141n4

Galileo 1–2, 5, 6, 14, 17, 35, 79, 84, 111; mathematics of 139n64; philosophy of mathematics 47

Gassendi, Pierre 3, 5, 9–10, 22, 28, 54–57, 71, 122, 123, 124; analysis of logic in schools 35–36; atomism of 34–39, 135–36n3; on atoms 38–39; Barrow and 97, 98, 102, 103, 104, 150n83; Berkeley and 75; Boyle and 135n2; constructivism of 42–46, 47; contributions to natural philosophy 33, 37, 39, 41–42; Descartes and 33–36, 38, 48–53, 56; Descartes and Gassendi on logic 55; development of natural philosophy 35–36; on empirical 'signs' 41–42; epistemology of 34, 36, 39–42, 43, 44, 45, 53, 68; friends and admirers 140n87; Hobbes and 62–66, 88; Hobbes on 141n6; on ideas, categories of 45; ignored as mathematical philosopher 126, 136n4; influences on thought of 35–36, 37; Locke and 66–69, 71; logic, 'canons' of 68; on logic, goals and limits of 43–44; mathematical philosophy of 48, 136n4; mathematics of 46–48; methodological differences with Descartes 52–53; natural philosophy, system of 26; Newton and 107, 108, 111; nominalism of 33, 39–48; objections to Descartes' metaphysics 51–52; philosophy of 8; physics, Gassendi's epistemology of 42; physics for 50–51; principles, understanding of 51–52; Pyrrhonism, interpretation of 18–19; Ramus' influence on 29, 30; scientific practitioner 33; on signs and mental operations 44–45; skepticism of human understanding 58–59; on space and matter 37–38; 'unification' according to 45; vacuum, existence and nature of 37–38; Wallis and 91

Geometrical Lectures (Barrow) 74, 96, 97, 101, 103

geometry 14, 16, 28, 46–47, 48, 88, 90, 112, 117; analytic 4, 55, 63, 64, 65, 66, 82, 86, 87, 90, 94–95, 109; Archimedean 5; Barrow's constructivist approach to 73, 74, 96–107; Berkeley's views on 80, 81, 82, 113–14, 116; classical 118–19; of Descartes 50–52, 62; Euclidean 5, 17, 86, 96, 98, 110; of Hobbes 62, 63, 66, 84, 109; Locke's preference for arithmetic over 71–72, 73; Maclaurin's view on 119–22; mathematical demonstrations in 8; mechanical 17, 62, 63, 96; Newton's use of 108–11; Plato's attitude to 13; principles of 9; symbolic 65; synthetic 94, 95; traditional 7, 64, 65, 83, 95, 115, 119; Wallis' view of 91–92

Geometry No Friend to Infidelity (Jurin, J.) 117

God 9, 20, 75, 87, 117, 121; absolute power of 9; atoms and 38–39; and causal structure of nature 104–5; creative power of 25; infinity of 38; language of 29; mathematical truths established by 50; and nature, composition of 39; perception and knowledge of existence of 51; in philosophy of Barrow 104–5, 106–7; space, matter and 37–38; universe, creation of 37

Goddu, André 24, 26

grammar 2, 21, 44, 55, 91, 130n4; abstraction as 47, 49, 100; of arithmetic 100; grammatical criticism, 16th Century 28–32; medieval and Renaissance 11; of signs 46, 49

Gregory, David 118, 152n129

Guicciardini, Niccolò 4, 109, 110, 151n92

Harriot, Thomas 7

Hatfield, Gary 6, 130n14

Heath, Thomas 131n16

Henricus Regius 141–42n8

Hill, Katherine 90, 95, 106, 147n6, 149n81

Hipparchus 17

History of Philosophy (Stanley, T.) 67, 68

Hobbes, Thomas 3, 5, 7, 9–10, 22, 26, 48, 55–67, 122–24; on abstraction 47; on algebra 65–67; analysis, view of 61; Barrow and 97, 98, 102, 103, 106; Berkeley and 117, 119; Berkeley's criticisms of 74; constructivism of 56–57, 62, 83; Descartes and 60; epistemological constructivism of 90; on Gassendi 141n6; Gassendi and 53, 54, 62–66, 75, 88; geometry of 62, 63, 66, 84–85, 109; Locke and 69, 70; Newton and 107, 108, 111; philosophy of mathematics 62–67; *Theory of Representation* 57–62; Wallis and 86, 89–90, 91

humanist scholarship 28–29, 35, 36, 52, 53, 123, 124

Hume, David 69, 122

Institutio Logica (Gassendi) 44

instrumentalism 116, 135n78, 152n123

De Interpretatione (Aristotle) 12, 15

Jardine, Lisa 134n73, 141n1

Jardine, Nicholas 30, 135n78

Jesseph, Douglas M. 3, 56, 59–60, 63, 66, 80, 113, 115–17, 120, 129n7, 141n3, 143n51, 146n118, 147n6, 153n131

Jones, Howard 37, 138n19

Jones, Richard Foster 90, 148n23

Joy, Lynn 35, 36, 47, 53

Jurin, James 117, 152n126

Kant, Immanuel 123

Kepler, Johannes 1, 5, 6, 14, 17, 30, 35, 104, 111, 121

Klein, Jacob 7, 49, 86–87, 130n19, 140n73

Koyré, Alexandre 3, 5, 6, 8, 36–37, 84, 128n2

Kroll, Richard W.F. 67, 68–69, 143n55
Kuhn, Thomas S. 111, 112, 151n101, 151n101/103, 151n103

language: Aristotle on 12–13; Berkeley's visual language of nature 76–80; epistemological questions about 12; epistemological status of 4–5; function of words and 12–13; of God 29; grammatical criticism, 16th Century 28–32; linguistic representation 21; mathematization and nature of 124–28; medieval and Renaissance grammar 11; 'mental symbols' 12; Ockham's approach to 23–24; philosophy of 12–20; roots of 20–21; Plato's concept of 78; Plato's interest in 12; Ramus' structural approach to 29–30; Sanches' linguistic concept of mental objects 30–32; *see also* grammar
Latin Grammar (Ramus) 29
Lectiones Geometricae (Barrow) 74, 96, 97, 101, 103
Lectiones Mathematicae (Barrow) 74, 97
Lectiones Opticae (Barrow) 74, 97
Leibniz, Gottfried W. 1, 10, 14, 37, 69, 95, 96, 109; calculus debate 112
Lennon, Thomas A. 2–3, 34, 128n5
Letter to Herodotus (Epicurus) 13
Leviathan (Hobbes) 57, 60, 61, 142n24
light, corporeality of 40–41
Linacre, Thomas 28
linguistic structuralism 146n110
Locke, John 29, 34, 35, 53, 55, 122, 124; abstractionism of 120; Barrow and 102; Berkeley and 74, 75, 77–78, 117, 120, 121; empiricist reconstruction of epistemology 67–73; epistemological constructivism of 85; epistemological position 69–73; the *Essay* 69–73; on ideas 144n73; Wallis and 90
logic 1–2, 8, 11, 15, 20, 28, 30, 34, 88, 90; Aristotelian 17, 19, 29, 35, 36, 132n37; Cicero on 44; epistemological skepticism in 122; experience and 27, 60; Gassendi and Descartes on 55; Gassendi's 'canons' of 68; limits of knowledge in 46; mathematical 6, 9, 97; medieval and Renaissance 11; nominalist beliefs in 106; Ockham's 22; philosophy of 44, 92; Ramus' formalist 29; scholastic 123, 133n40, 139n47; structural aspects of 21
Logic (Port-Royal logicians) 55
Logic (Ramus) 29
Lombard, Peter 25
Lucretius 33, 38

McGuire, J.E. 108, 150n87, 151n91
Maclaurin, Colin 118–22, 152n129, 153n130
Mahoney, Michael S. 4, 6–7, 96, 130n17, 148n43, 150n85
Malet, Antoni 104, 105–6, 149n71
Mancosu, Paolo 3, 7, 130n20, 153n1
Mathematical Lectures (Barrow) 74, 97
Mathematical Miscellanies (Berkeley) 74
mathematical philosophy 122, 124; beliefs during Scientific Revolution and 3; of Berkeley 74, 80–82, 121–22, 152n116; constructivism in 126–27; of Descartes 47; development of 4, 84, 86; emergence of modern 10, 84, 86; epistemological uncertainty in 112; formalist approach to 5; of Galileo 47; of Gassendi 48, 136n4; of Hobbes 62–67; meanings of construction and 84–85; Newton's contribution to 111; nominalism in 126–27; ontological approaches to 84; ontological uncertainty in 112
mathematical physics 1, 6, 10, 19; Newtonian science and 122
mathematical sciences 3, 4, 5, 6, 85, 120, 129n8
mathematics: in antiquity 12–20; Aristotle's philosophy of 15–16; calculus debate 111–22; Cartesian mathematical realism 34; constructivism and mathematization 84–87; counterfactual mathematical universes 9; existence of mathematical objects 16; foundations of 47; of Galileo 139n64; of Gassendi 46–48; language, mathematization and nature of 124–28; linguistic representation and 8; logical representation and 8; mathematical Neoplatonism 17–18; mathematical truths established by God 50; metaphysics of 5–6; as mode of representation 13–14; objections to metaphysical basis for 6; Ockham's 'pragmatic conception' of 24; ontological basis for demonstrations 6–7; Plato's philosophy of 14–15; Pythagoras' mathematical realism 19; representative value of 7; *see also* algebra; analysis; arithmetic; atomism; constructivism; empiricism; epistemology; geometry; grammar; language; logic; metaphysics;

nominalism; number; philosophy; physics; science; skepticism
mathematization 1, 9, 106; Cartesian 34; constructivism and 84–87; epistemology of 2, 6–7; and language, nature of 1–10, 124–28; of natural phenomena 7, 95–96; origins of 5; Scientific Revolution and 8
Mathesis Universalis (Wallis) 74, 87, 92
Meditations on First Philosophy (Descartes) 33, 48, 51, 56, 60, 68, 70
Melanchthon, Philip 28
Meno (Plato) 14
Mersenne, Marin 47, 50, 54, 56
metaphysics 2, 59, 88, 94, 100, 112, 120–21, 122; of Aristotle 15–17; Descartes' positions 51; Descartes' questioning 50; existence, metaphysical questions concerning 15–16; linguistic approach to representation 21; of mathematization 5; medieval and Renaissance 11; metaphysical being 91; metaphysical essence 108; metaphysical falseness 93; metaphysical foundation of mathematics 3, 6; metaphysical realism 21, 53; Newton's questioning 110; nominalism and metaphysical realism 53; nominalist metaphysics 9, 10, 26; Platonist 14, 15, 21
Metaphysics (Aristotle) 15
Michael, Fred and Emily 68–69, 144n60
Mondiak, Deborah K.W. 15, 131n6, 132n25
More, Henry 17
Morrow, Glenn R. 17, 132n27
motion, impetus and 63

natural philosophy 18, 20, 26–27, 31, 44, 55, 69, 121; conceptual foundations 3–4; constructivism in 17th Century 85; development of mathematical 123; experimental 2, 87, 95; fundamental aspects of 34; Gassendi's contributions to 33, 37, 39, 41–42; Gassendi's development of 35–36; Hobbes' vision for 60–61; language of 96; as logical reconstruction of nature 44; mathematical demonstrations in 4–5, 6–8, 85–86, 92, 105, 112; metaphysical foundations 3–4; 'narrative' approach to 148n34; Newton's system of 67, 83–84, 111, 122; Wallis' practice of 88
The Nature of Truth, its Union and Unity with the Soule (Brooke, R.) 87
Neoplatonism 14, 17
'Newcastle Circle' 56
Newton, Isaac 1, 3–5, 7, 10, 37, 54, 55, 107–11, 124, 126; algebraic notation 109; Barrow and 84, 85, 92, 104; Berkeley and 74, 116, 117, 118, 119, 121; calculus debate 112–15; calculus of fluxions 63, 112, 113; constructivism of 107, 110; epistemology of 111; Gassendi and 33–35, 38, 107, 108, 111; geometry, use of 108–11; Hobbes and 62, 107, 108, 111; Locke and 69; mathematical philosophy, contribution to 111; mathematical physics, Newtonian science and 122; mathematized physics of 87, 112, 122; metaphysics, questioning 110; natural philosophy, system of 67, 83–84, 111, 122; Newtonian calculus 82; Newtonian science 118, 122; Newtonianism and actual beliefs of 122; philosophy of 107, 112, 118, 122, 135n2; problematic and fascinating character 83–84, 87; unpublished mathematical papers 109–10; Wallis and 95, 96
Newtonianism 118, 122
nominalism 2, 3, 5, 8, 30, 57, 87, 107, 123, 124; atomism and 27–28; Barrow's nominalistic view on arithmetic 98–101, 103, 106; Berkeley's 67, 69, 76, 81; core value of epistemology 56–57; early modern thought and 128n6; and empirical skepticism of Berkeley 122–23; and epistemology, relationship between 83; epistemology of 9–10; of Gassendi 33, 39–48; Hobbes' extreme nominalism 56, 63, 66, 106; linguistic 82; Locke's empiricism and 67, 69; logic, nominalist beliefs in 106; mathematical constructivism and 86, 125–26; in mathematical philosophy 126–27; medieval nominalists 20–28; and metaphysical realism 53; in Middle Ages 20–28; nominalist epistemology 18, 26, 33, 87, 125; nominalist metaphysics 9, 10, 26; nominalist philosophy of representation 10; nominalist/voluntarist tradition 9; Ockham and 22–23; ontological implications 133–34n56; realism and 83–84, 102; 17th century nominalists 19–20; skepticism in nominalist metaphysics 26; theological implications 133–34n56
Norton, David Fate 67, 143n54
number 14, 16, 19, 61, 71, 80–81, 85, 89; basic idea of 72; clear and distinct idea

of 120; concept of 86–87, 92, 93; existence of 100; irrational numbers 93, 125; nature of 90–91; negative 94; as 'note' or 'sign' 74; Ockham's treatment of 24–25; ontological status of 24; physical quantity, freedom from connection to 49

Ockham, William of 9, 20, 22–25, 26–27, 30, 45, 56, 75, 133n45, 136n6; linguistic conception of mental processes 23–24; nominalism and 22–26; 'pragmatic conception' of mathematics 24–25
Oldenberg, Henry 96, 148n42, 151n100
Ong, Walter 29
Opera (Gassendi) 35, 36, 43, 44, 56, 68, 137n13
Opera Mathematica (Wallis) 147n10
Optical Lectures (Barrow) 74, 97
Oresme, Nicole 9, 22
Osler, Margaret J. 136n6, 149n73
Oughtred, William 7, 108
Outlines of Skepticism (Sextus Empiricus) 18

Padley, G.A. 29, 56, 134n71, 141n2, 142n25
Panaccio, Claude 23
Parmenides 12
Pemberton, Henry 118, 152n129
Phaedo (Plato) 14
Philosophae Naturalis Principia Mathematica (Newton) 4, 55, 107, 108, 109, 111, 112, 118, 121, 151n100
Philosophical Transactions (Royal Society) 87
philosophy: analytical methods in mathematics and 7–8; of Aristotle 21, 29, 111–12; Berkeley's philosophy of mind 75–76; Cartesian mechanical philosophy 111; Christianized Aristotelian philosophy 39; Christianized Platonic philosophy 39; of Descartes 47, 69, 95, 111, 140n76; of Epicurus 37; Galileo's philosophy of mathematics 47; of Gassendi 8; God in philosophy of Barrow 104–5, 106–7; of language 12–21; of logic 44, 92; of Newton 107, 112, 118, 122, 135n2; nominalist philosophy of representation 10; scholastic philosophy, Gassendi's critique of 35–36; of science 53, 57; secondary library on philosophy of Berkeley 145n94; skeptical philosophy 56; task of 40–41; traditions in history of 2–3; *see also* mathematical philosophy; natural philosophy
physics 2, 6, 10, 19, 49, 57, 88, 111; of atoms 34–39; Berkeley's views on 115–16; Gassendi's epistemology of 42; geometry and 62; Hobbes' mechanical physics 63; mathematical physics of Descartes 1; mathematical understanding of 95; mathematics and, relationship between 50–51; of movement 27; Newton's mathematized physics 87, 112, 122; and 'pure' mathematics 4, 52; qualitative physics of Aristotle 5
Physics (Aristotle) 15, 26
Piccolomini 7, 126, 139n60
Pico della Mirandola, Giovanni 6
Plato 11–18, 20, 28, 31, 45; concept of language 78; doctrine of forms 30; doctrine of 'recollection' or reminiscence 50, 70; idealism of 8, 14, 30; metaphysics of 6, 21; realism of 22
Platonism 14, 16, 52–53, 88–89, 97, 107, 108, 124, 125
Popkin, Richard H. 18, 132n30
positivism: anti-positivism 6; pervasiveness in early 20th century 128n1
Posterior Analytics (Aristotle) 13, 15
du Prat, Abraham 56
Principles of Philosophy (Descartes) 49
Proclus 11, 17, 18; *Commentary* 132n29
Prodicus 12
Protagoras 12
Ptolemy 104; systems of 47–48
Pycior, Helena M. 7–8, 67, 74, 90, 97, 106, 116, 117, 130n21, 143n45, 145n91, 147n6, 149–50n82, 149n54
Pyrrho of Ellis 18, 36, 43; relativism of 19; skepticism of 11, 17, 18–19, 36, 43
Pythagoras 31; mathematical realism 19; metaphysics 6
Pythagoreans 16

Quaestiones quaedam philosophicae (Newton) 109, 150n87
Quaestro de certitudine mathematicarum (Piccolomini) 7, 126
Quod Nihil Sciatur (Sanches) 30, 139n52
Quodlibertal Questions (William of Ockham) 24, 25

Ramus, Petrus 28, 29, 44, 135n76; formalist logic of 29–30
Reiss, Timothy J. 29, 134n72, 135n77
Rimini, Gregory of 137n17

Rogers, G.A.J. 144n64
Royal Society of London 54, 69, 87, 141n1
Rudiments (Ramus) 29

Sanches, Francisco 28, 30–32, 135n80
Scaliger, J.C. 29
scholastic philosophy 17, 20, 21; Gassendi's critique of 35–36
scholasticism 132n37
science 8, 31–32, 34–35, 55, 92, 98, 114, 121; commentators in antiquity on 18; concerns of 'old' science 123; Gassendi in touch with 35–36; geometry as 'mechanical' science 63; history of 127; Hobbes' positive approach to 64–66; human ingenuity and 8–9; mathematics as 13, 15; method of 58, 61; modern Western science 1, 5, 7; Newtonian 118, 122; philosophy of 53, 57; physics as science of approximation 116; representation of 54; of signs 74, 112, 117; slogans of 'new' science 123; *see also* algebra; geometry; mathematical sciences; natural philosophy; physics
Scientific Revolution 2, 3–4, 6, 8, 88, 126, 127; Aristotle's physical ontology, rejection of 17; constructivism of 85; empiricist inclinations 149n54; experimental spirit 14; mathematical sciences, vital change in 4; mathematics as language 5; 'mathematization of nature,' strategies for 125; notion of 129n9; origins of 132n37; re-evaluations of the 34; rejection of Aristotle's physical ontology 17
Sentences (Lombard) 25
Sextus Empiricus 17, 18, 19, 30, 36, 43
Shapin, Steven 148n34
Six Lessons to the Professors of Mathematics (Hobbes) 65
skepticism 30, 83, 104–5, 106, 115–16; Academic skepticism 11, 18, 30, 138n32; classical skepticism, 16th Century 28–32; constructivist skepticism 116; Gassendi, of human understanding 58–59; *The History of Skepticism* (Popkin, R.H.) 132n30; nominalism and empirical skepticism of Berkeley 122–23; potential in nominalist metaphysics 26; of Pyrrho of Ellis 11, 17, 18–19, 36, 43; radical skepticism 88, 105; skeptical philosophy 56; solipsistic skepticism 75
sociology 8–9
Socrates 12, 14
Sorbière, Samuel 56, 140n87, 141–42n8
specialization 4
Spinoza, Benedict 17
Stanley, Thomas 67, 68, 144n61
Stoics 11, 13; dictum of *quod nihil sciatur* 45, 137n17
Strong, Edward W. 6, 130n15
Syntagma Philosophicum (Gassendi) 35, 36, 37, 39–40, 41, 67, 68

Tamny, Martin 108, 150n87, 151n91
Theaetetus 14
Theory of Vision, or Visual Language (Berkeley) 76, 79, 117
Thomas Aquinas 11, 17, 20, 21, 28, 29; on mathematics 133n40
Timaeus (Plato) 13
A Treatise Concerning the Principles of Human Knowledge (Berkeley) 55, 75–76, 117
A Treatise of Fluxions (Maclaurin, C.) 118, 121
Truth Tried: Animadversions on the Nature of Truth, its Union and Unity with the Soule (Wallis) 87, 90, 91, 92, 93

Universal Treatise (Nicholas of Autrecourt) 25–26

vacuum 34, 150n83; existence and nature of 37–38
Valla, Lorenzo 28, 29
Vieta, François 49, 86; symbolic algebra of 86, 126
A View of Sir Isaac Newton's Philosophy (Pemberton, H.) 118

Wallis, John 7, 63–65, 74, 83, 86, 87–96, 97–101, 106, 121, 123–24; algebra, approach to 92–96; arithmetic, approach to 87, 93–95; constructivism of 125; epistemology of 95; geometry, view of 91–92; indivisibles, method of 108–9; natural philosophy, practice of 88; *Opera Mathematica* 147n10; theological subjects 147n11
Ward, Seth 55
Webster, John 55
Westfall, Richard 36–37, 138n21
Whiteside, D.T. 148n45
Wilkins, John 55
Wyclif, John 22

Zamberti, Bartolomeo 6
Zeno 13; paradoxes of 26–27

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